Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 47, No. 2, pp.22–33, 2003 *UDC 517.968*

RIESZ SUMMABILITY OF DECOMPOSITIONS BY EIGENFUNCTIONS OF INTEGRAL OPERATORS

A.P. Gurevich and A.P. Khromov

The question on an equiconvergence by eigen- and adjoined functions of the integral operator $Af = \int_0^1 A(x,t)f(t)dt$ and into the ordinary trigonometric Fourier series was investigated in [1]. The equiconvergence was established under the following assumptions:

a) the derivatives $A_{x^s t^j}(x,t) = \frac{\partial^{s+j}}{\partial x^s \partial t^j} A(x,t)$, $s, j = 0, \dots, n$, are continuous for $t \le x$ and $t \ge x$;
b) $P_{sj}(t) = \Delta A_{x^s t^j}(x,t)|_{x=t} = A_{x^s t^j}(x,t)|_{x=t+0} - A_{x^s t^j}(x,t)|_{x=t-0} \in C^{n-1-j}[0,1]$, $j=0,\dots,n-1$, $s = 0,\dots,n$;
c) A^{-1} exists;
d) $\Delta A_{x^s}(x,t)|_{x=t} = \delta_{s,n-1}$, $s = 0,\dots,n$ ($\delta_{s,n-1}$ is the Kronecker symbol).

It was proved that condition c) is necessary for the equiconvergence, conditions a) and b) are exact, while condition d) says about the canonical form of the integral operator for which the considered equiconvergence takes place. Under the fulfillment of conditions a)–d) A^{-1} represents the following integral-differential operator:

$$A^{-1}y = (E+N)(y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y), \tag{1}$$

$$V_j(y) = U_j(y) - (y, \varphi_j) = 0, \quad j = 1,\dots,n. \tag{2}$$

Here E is the unit operator, $Nf = \int_0^1 N(x,t)f(t)dt$, where the kernel $N(x,t)$ is continuous for $t \le x$ and $t \ge x$, $a_1, \dots, a_n$ are some constants,

$U_j(y) = \sum_{\mu=0}^{\sigma_j} (a_{j\mu} y^{(\mu)}(0) + b_{j\mu} y^{(\mu)}(1))$, $j = 1,\dots,n$,
$|a_{j,\sigma_j}| + |b_{j,\sigma_j}| > 0$, $n-1 \ge \sigma_1 \ge \dots \ge \sigma_n \ge 0$, $\sigma_j > \sigma_{j+2}$,
$(y, \varphi_j) = \int_0^1 y(x)\varphi_j(x)dx$, $\varphi_j(x) \in C[0,1]$.

For the equiconvergence one additional condition is required:

e) $U_j(y)$, $j = 1,\dots,n$, are Birkhoff regular (see [2], p. 66).

In the present article we investigate the convergence of the Riesz means

$$-\frac{1}{2\pi i}\int_{|\lambda|=r} g(\lambda,r) R_\lambda f \, d\lambda.$$

Here $R_\lambda = (E - \lambda A)^{-1}A$ is the Fredholm resolvent, while $g(\lambda, r)$ satisfies the following conditions:

f) $g(\lambda,r)$ is continuous with respect to λ in the disk $|\lambda| \le r$ and is analytic with respect to λ in the disk $|\lambda| < r$ for any $r > 0$,

The work was supported by the Russian Foundation for Basic Research (project no. 00-01-00075), the Program "Leading Scientific Schools" (project no. 00-15-96123), and the Program "Universities of Russia".