ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 5, pp. 9–22. © Allerton Press, Inc., 2012.
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Some Estimations for the Least Degree of Identities of Subspaces $M_1^{(m,k)}(F)$ of the Matrix Superalgebra $M^{(m,k)}(F)$

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Received January 11, 2011

Abstract—We estimate the least degree of identities of subspaces $M_1^{(m,k)}(F)$ of the matrix superalgebra $M^{(m,k)}(F)$ over the field F for arbitrary m and k. For subspaces $M_1^{(m,1)}(F)$ ($m \geq 1$) and $M_1^{(2,2)}(F)$ we obtain concrete minimal identities.

DOI: 10.3103/S1066369X12050027

Keywords and phrases: *T-ideal, polynomial identity, matrix superalgebra.*

INTRODUCTION

Let F be an arbitrary field; let $F\{Z\}$ be a free associative algebra generated by a countable set $Z = \{z_i\}_{i\in\mathbf{N}}$; let m and k be arbitrary natural numbers; let $M_{m+k}(F)$ be the algebra of $(m+k)\times(m+k)$-matrices over the field F; let $M^{(m,k)}(F) = (M_{m+k}(F), M_0^{(m,k)}(F), M_1^{(m,k)}(F))$ be a matrix superalgebra graded by subspaces

$$M_0^{(m,k)}(F) = \left\{ \begin{pmatrix} C_{m\times m}(F) & 0_{m\times k} \\ 0_{k\times m} & D_{k\times k}(F) \end{pmatrix} \right\}, \quad M_1^{(m,k)}(F) = \left\{ \begin{pmatrix} 0_{m\times m} & B_{m\times k}(F) \\ A_{k\times m}(F) & 0_{k\times k} \end{pmatrix} \right\},$$

$M_2^{(m,k)}(F) = M_0^{(m,k)}(F) \oplus M_1^{(m,k)}(F)$, and let S_n be a symmetric group of degree n.

Definition 1. A polynomial $f(z_{i_1}, \ldots, z_{i_n}) \in F\{Z\}$ is called a polynomial identity of the subspace $M_i^{(m,k)}(F)$, where $i = 0, 1, 2$, if any homomorphism $\varphi \in \mathrm{Hom}_F(F\{Z\}, M_{m+k}(F))$ such that $\varphi(L(Z)) \subseteq M_i^{(m,k)}(F)$, where $L(Z)$ is a linear span of the set Z, satisfies the equality $\varphi(f(z_{i_1}, \ldots, z_{i_n}))=0$.

One can easily notice that the set of all polynomial identities of the subspace $M_i^{(m,k)}(F)$, where $i = 0, 1, 2$ (in what follows we assume that i equals 0, 1, or 2), forms a two-sided ideal of the algebra $F\{Z\}$, which is called the ideal of identities of the subspace $M_i^{(m,k)}(F)$ and is denoted by $T[M_i^{(m,k)}(F), M_{m+k}(F)]$. We use a shorter denotation, namely, $T[M_i^{(m,k)}(F)]$.

The description of the basis of identities of the matrix algebra $M_n(F)$ for arbitrary n and F is a sophisticated problem; it is not solved yet ([1], problem 117). At present time T-ideals of the following algebras are described: $M_2(F_1)$, where F_1 is a field of zero characteristic, and $M_n(F_2)$, where $n = 2, 3, 4$, while F_2 is a finite field (see references in [2]). In addition, from [2] (pp. 66–67) it follows that if $\operatorname{char} F = 0$, then the ideal of identities $T[M_n(F)]$ is finitely basable.

Since the dimension of the algebra $M_{m+k}(F)$ is finite, in view of the linearization process, the set $T[M_i^{(m,k)}(F)] \neq \{0\}$ and contains at least one nonzero polylinear identity $g_i(z_1, \ldots, z_{q_i}) = \sum\limits_{\sigma\in S_{q_i}} \alpha_{i\sigma} z_{\sigma(1)} \ldots z_{\sigma(q_i)}$.

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