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# Ideal Extensions of Lattices*

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**Abstract**—Following the well-known Schreier extension of groups, the (ideal) extension of semigroups (without order) have been first considered by A. H. Clifford in Trans. Amer. Math. Soc. **68** (1950), with a detailed exposition of the theory in the monographs of Clifford–Preston and Petrich. The main theorem of the ideal extensions of ordered semigroups has been considered by Kehayopulu and Tsingelis in Comm. Algebra **31** (2003). It is natural to examine the same problem for lattices. Following the ideal extensions of ordered semigroups, in this paper we give the main theorem of the ideal extensions of lattices. Exactly as in the case of semigroups (ordered semigroups), we approach the problem using translations. We start with a lattice $L$ and a lattice $K$ having a least element, and construct (all) the lattices $V$ which have an ideal $L'$ which is isomorphic to $L$ and the Rees quotient $V|L'$ is isomorphic to $K$. Conversely, we prove that each lattice which is an extension of $L$ by $K$ can be so constructed. An illustrative example is given at the end.

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## INTRODUCTION

The extension problem for groups is as follows: Given two groups $H$ and $K$ construct all groups $G$ which have a normal subgroup $N$ which is isomorphic to $H$ and the quotient $G/N$ of $G$ by $N$ is isomorphic to $K$. $G$ is the well-known Schreier extension (or simply the extension) of $H$ by $K$. Following the Schreier extension of groups, the ideal extensions of semigroups have been considered by A. H. Clifford in [1]. A detailed exposition of the ideal extensions of semigroups can be found in [2, 3]. The main theorem of the ideal extension of semigroups is as follows: Given a semigroup $S$ and a semigroup $Q$ with zero such that $S \cap Q^* = \emptyset$ (where $Q^* = Q \setminus \{0\}$), construct all the semigroups $V$ which have an ideal $S'$ which is isomorphic to $S$ and the Rees quotient $V|S'$ is isomorphic to $Q$. To avoid confusion, for the Rees quotient we use the notation $V|S$ instead of the usual one $V/S$. Extensions of weakly reductive semigroups, strict and pure extensions, retract extensions, dense extensions, equivalent extensions have been also considered in [3]. Ideal extensions of totally ordered semigroups have been studied in [4, 5], those of topological semigroups in [6, 7]. We are often interested in building more complex semigroups, lattices, ordered sets, ordered or topological semigroups and this can be sometimes achieved by constructing the ideal extensions. Ideal extensions of ordered sets have been considered in [8]. The retract and the equivalent extensions of ordered sets have been considered in [9, 10]. For the ideal extensions of ordered semigroups we refer to [11]. As in semigroups (without order), for ordered semigroups we approach the problem using left and right translations. We start with an ordered semigroup $S$ and an ordered semigroup $Q$ with zero (to avoid trivialities $Q$ must have at least one nonzero element) such that $S \cap Q^* = \emptyset$ and construct all the ordered semigroups $V$ having an ideal $S'$ which is isomorphic to $S$ and the Rees quotient $V|S'$ is isomorphic to $Q$. Conversely, we prove that each ordered semigroup which is an extension of $S$ by $Q$ can be so constructed. Since the problem of the ideal extensions has been already considered for semigroups, for ordered semigroups, and ordered sets, it is quite natural to examine the same problem for lattices. The aim of this paper is to construct the ideal extensions of lattices. We give the main theorem of such extensions and an illustrative example

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