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APPROXIMATION BY ELEMENTS OF WEDGE WITH REGARD FOR VALUES OF APPROXIMATING ELEMENTS

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Introduction

In [1]–[5] the theory was developed for approximating processes in which not only the convergence of approximating aggregates to an element of approximation was taken into account but also values of these approximating aggregates. In addition, the values of approximating elements are measured, generally speaking, in a metric which differs from that the approximation is carried out in. The most complete treatment of these questions can be found in [4] and [5] (see § 2 there). In [6] this theory was extended to the case where the approximation must be done with a beforehand given rate. In all these works the approximation was carried out by means of elements of a certain linear subspace. The above theory possessed sufficiently numerous applications and, in particular, turned out to be useful in the studying of the concept of analytic capacity of sets and its various modifications which are important for the theory of analytic functions. These questions are treated in detail in [5], where an ample bibliography can be found. Other applications of the mentioned theory are given in [4], where also detailed references can be found. In this article the theory of approximation with regard for approximating elements is extended to the case where the approximation is carried out by elements of a certain wedge selected in the space (not in a subspace, as earlier).

A stimulus to the consideration of similar problems lies in the possibility to study on this basis some useful modifications of the concept of analytic capacity. These modifications will be treated in detail in another eventual article whose starting point should be the relations from Section 4 of this article.

1. Duality theorem by Garkavi in case of a convex functional which is not obligatory symmetric

Consider a real linear space X and assume that X' is a space of all linear functionals over X. In X a convex nonnegative functional $r(x)$ is given, $r(x) \geq 0$ (the functional $r(x)$ is convex if $\forall x_1, x_2$ $r(x_1 + x_2) \leq r(x_1) + r(x_2)$ and $\forall \alpha$ $r(\alpha x) = \alpha r(x)$). Assume that a convex set $E \subset X$ is also given and for a certain element $\omega \in X$ we consider the problem of best approximation

$$\inf_{x \in E} r(\omega - x). \tag{1.1}$$

In the case where X is a normed space while $r(x)$ is the norm in X, problem (1.1) is the standard problem of best approximation. In the present case it is complicated by the fact that $r(x)$ is not assumed to be symmetric, i.e., in general, $r(-x) \neq r(x)$. Our nearest aim is to obtain dual expressions for (1.1). In case E is a linear subspace of the normed space X and $r(x)$ the norm in X, the dual relations were obtained in well-known classical works by M.G. Kreĭn (see [7]) and

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