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ALGORITHMS IN THE METHOD OF CENTERS WITH APPROXIMATION OF THE SET OF ADMISSIBLE SOLUTIONS

Ya.I. Zabotin and A.A. Andrianova

In the application of iteration methods for solving problems of the mathematical programming, which, as a rule, are based on some heuristic arguments about attainability of the prescribed accuracy of a solution, one should stop the computational process, because theoretical criteria of optimality are fulfilled only in the sought-for point of the optimum. Therefore, for a researcher which has to carry out the computation in applying a certain method of optimization it is of importance to have not only optimality criteria, but also some easily verifiable conditions whose fulfillment ensures that the desired solution will be attained within an a priori given finite number of iterations. This article is devoted to the development of algorithms which produce conditions of that type. These conditions are obtained for methods of both interior and exterior centers (see [1], pp. 83–90) at the expense of the use of a neighborhood in the algorithms and, respectively, a subset of the set of admissible solutions. It is assumed that auxiliary problems of unconditional minimization can be solved exactly.

The problem of an interruption of iteration process was considered, for example, in [2]–[6]. In [2], a parameterization of the distance functions was suggested and it was proved that, in principle, due to the choice of the values of the parameter one can attain the prescribed accuracy within a single iteration process of the minimization of the auxiliary maximum function. Nevertheless, since it was impossible to indicate the values of the parameters, in [3]–[5] a procedure of parameter adaptation was carried out. However, this procedure possessed an iterative character. Finally, in contrast to [6], here we develop criteria which allow us to attain the prescribed accuracy within a finite number of iterations by means of a one-sided (but not two-sided) process of approximations.

1. Properties of the maximum function

In this Section we give auxiliary assertions for additively parameterized maximum function in a form which will be necessary in what follows.

Let the functions $\varphi(x)$, $\psi(x)$ be defined and continuous in an n-dimensional Euclidean space R_n, $G = \{x : x \in R_n,\ \psi(x) \le 0\}$, $G \ne \emptyset$, $x^* \in \operatorname{Arg\,min}\{\varphi(x),\ x \in G\}$, $\varphi^* = \varphi(x^*)$.

As in [6], we denote by M a set of functions defined in R_n, for which every local minimum is absolute. It is assumed that $\psi \in M$.

Put

$$\Phi(x,t,\gamma) = \max\{\varphi(x) - t, \psi(x) - \gamma\}, \quad t, \gamma \text{ are constants},$$
$$Z = \operatorname{Arg\,min}\{\Phi(x,t,\gamma),\ x \in R_n\}.$$

Lemma 1. *Assume that the parameters t, γ are fixed in such a way that a point $z \in Z$ exists, $z \notin G$. Then*

1. $\varphi(x) - t > \psi(x) - \gamma\ \ \forall x \in G$;