

Completeness of Eigenfunctions of One Class of Pencils of Differential Operators with Constant Coefficients

V. S. Rykhlov^{1*}

¹Saratov State University, ul. Astrakhanskaya 83, Saratov, 410012 Russia

Received April 2, 2007

Abstract—In spaces of functions square summable on finite intervals we study the simple completeness of the system of eigen- and associated functions for the pencil of ordinary differential operators generated by a differential expression with constant coefficients (we assume that the roots of the corresponding characteristic equation lie on one and the same ray) and specific semisplitting homogeneous boundary value conditions.

DOI: 10.3103/S1066369X09060061

Key words and phrases: *multiple completeness, simple completeness, eigen- and associated functions, pencil of ordinary differential operators.*

1. PROBLEM DEFINITION

In the space $L_2[0, 1]$ we consider a pencil of ordinary differential operators $L(\lambda)$ generated by a homogeneous differential expression of the n th order with constant coefficients

$$\ell(y, \lambda) := \sum_{s+k=n} p_{sk} \lambda^s y^{(k)}, \quad p_{sk} \in \mathbb{C}, \quad p_{0n} \neq 0, \quad (1)$$

and linearly independent two-point normalized homogeneous boundary conditions that have a special structure

$$\begin{aligned} U_j(y, \lambda) &:= \sum_{s+k=\varkappa_j} \lambda^s \alpha_{j sk} y^{(k)}(0) = 0, \quad j = \overline{1, n-1}, \\ U_n(y, \lambda) &:= \sum_{s+k=\varkappa_n} \lambda^s \left(\alpha_{n sk} y^{(k)}(0) + \beta_{n sk} y^{(k)}(1) \right) = 0, \end{aligned} \quad (2)$$

where $\lambda \in \mathbb{C}$ is a spectral parameter, $\alpha_{j sk}, \beta_{n sk} \in \mathbb{C}$, $\varkappa_j \in \{0, 1, \dots, n-1\}$ is the order of the j th boundary condition.

Let roots $\{\omega_j\}_1^n$ of the characteristic equation

$$\sum_{s+k=n} p_{sk} \omega^k = 0$$

be nonzero and different; assume that they belong to one and the same ray that goes out of the origin of coordinates. Without loss of generality we assume that

$$0 < \omega_1 < \omega_2 < \dots < \omega_n. \quad (3)$$

We want to establish conditions for parameters of the pencil $L(\lambda)$ under which the m -fold ($1 \leq m \leq n$) completeness of the system of eigen- and associated functions (e. a. f.) of this pencil in

*E-mail: RykhlovVS@info.sgu.ru.