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Equivalence of Curves with Respect to the Action of the Symplectic Group

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Abstract—We establish an equivalence criterion for finite systems of curves with respect to the action of the symplectic group.

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One of important problems in differential geometry is finding conditions that guarantee equivalence of curves with respect to an action of a particular algebraic group G. Recently, in solving this problem, methods of the theory of invariants are widely used. For this purpose, differential fields of all differential G-invariant rational functions for a curve are studied, finite rational bases of these fields are described, and, with the use of them, criteria of equivalence of curves are established. Such an approach was used in [1] in solving the problem on equivalence of curves with respect to an action of the semidirect product $R^n \triangleleft SL(R,n)$ of groups R^n and $SL(R,n)$. In [2], with the use of this method, the problem on equivalence of paths was solved in the case of action of the symplectic group $\mathrm{Sp}(2n,R)$. The question on equivalence of paths for actions of other particular groups was studied in [3].

In this paper, we solve the problem on equivalence of curves with respect to the action of the symplectic group. We use the terminology and notation from [1–3].

Consider the n-dimensional vector space $V = R^n$ over the field of real numbers R. Elements of V will be represented as n-dimensional vector-rows. By an $\Im$-path $x(t)$ we mean an infinitely differentiable mapping x from a segment $\Im = [a,b] \subset R$ to V. In terms of coordinates, such a path is given by a vector-function $x(t) = (x_1(t),\dots,x_n(t))$, where $t \in \Im$ and $(x_1,\dots,x_n)$ are Cartesian coordinates in V. The image of a segment $\Im$ under a mapping $x : t \to x(t)$ is called the carrier of $x(t)$, we denote it by $\widetilde{x} = \{(x_1(t),\dots,x_n(t)) \in R^n,\ t \in \Im\}$. An $\Im$-path $x(t)$ is called simple if $x(t)$ is an injective mapping from $\Im$ to V.

Let $SL(R,n)$ be the group of all invertible linear transformations of V, and let G be a subgroup of $SL(R,n)$. We will consider the right action $(g,x) \to xg$ of G on V, i.e., the standard multiplication of a row by a matrix.

Two $\Im$-paths $x(t)$ and $y(t)$ are called G-equivalent if an element $g \in G$ exists such that $x(t)g = y(t)$ for any $t \in \Im$.

The rth order derivative of an $\Im$-path $x(t) = (x_1(t),\dots,x_n(t))$ is defined to be the vector $\overset{(r)}{x}(t) = \big(\overset{(r)}{x_1}(t),\dots,\overset{(r)}{x_n}(t)\big)$, where $\overset{(r)}{x_i}(t)$ is the rth derivative of the coordinate function $x_i(t)$, $i = 1,\dots,n$.

For each $\Im$-path $x(t)$, consider the $n \times n$-matrix $M(x)$ whose r-row consists of the coordinates of the vector $\overset{(r-1)}{x}$, $r = 1,\dots,n$. In what follows we consider only simple $\Im$-paths $x(t)$ for which the determinant $\det M(x)(t)$ is not zero for all $t \in \Im$ (in this case $x(t)$ is said to be a regular path). Note that two $\Im$-paths $x(t)$ and $y(t)$ are G-equivalent if and only if $M(x) = M(x)g$ for some $g \in G$.

Two paths $x : \Im_1 \to V$ and $y : \Im_2 \to V$ are said to be D-equivalent if a C^∞-diffeomorphism $\varphi : \Im_2 \to \Im_1$ exists such that $y(t) = x(\varphi(t))$ and $\varphi'(t) \neq 0$ for all $t \in \Im_2$.

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