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A DIRECT SOLUTION METHOD FOR A SINGULAR INTEGRAL DIFFERENTIAL EQUATION ON A SEGMENT OF THE REAL AXIS

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Introduction

Various applied problems imply the solution of the singular integral differential equation

$$Kx \equiv x'(t) + a(t)x(t) + \frac{b(t)}{\rho(t)} \int_{-1}^{+1} \frac{\rho(\tau)x(\tau)}{\tau - t} d\tau = f(t), \quad -1 < t < 1, \tag{1}$$

under the initial condition

$$x(-1) = 0, \tag{2}$$

where $\rho(t) = (1-t)^\alpha(1+t)^\beta$, $-1 < \alpha, \beta < 1$, is the Jacoby weight, $a(t), f(t) \in L_{2\rho}[-1,1]$, and $b(t) \in C[-1,1]$, and a singular integral is understood as the Cauchy principal value (see [1], [2]).

One can solve the Cauchy problem (1)–(2) exactly only in rare particular cases. Therefore many approximate methods are used to solve this problem (see, e.g., [3], [4] and references there). In [4], the solution of problem (1)–(2) is considered for a special case, when $\rho(t) = \frac{1}{\sqrt{1-t^2}}$ are the Chebyshev weights of the I kind; in [5], the case $\rho(t) \equiv 1$ is studied. In this paper, we generalize the mentioned particular cases, solving problem (1)–(2) for the Jacoby weight function.

We solve the Cauchy problem (1)–(2) by the least square method. We describe the scheme of the method and substantiate the convergence of the sequence of approximate solutions obtained by means of the least square method. We essentially use the related results of the monographs [3] and [4]. The obtained result is announced in [6].

1. Main results

We seek for an approximate solution of problem (1)-(2) in the form

$$x_n(t) = \sum_{k=1}^{n} \alpha_k [P_k^{(\alpha,\beta)}(t) - P_k^{(\alpha,\beta)}(-1)], \quad n \in \mathbb{N}, \tag{3}$$

where $\{P_k^{(\alpha,\beta)}(t)\}$ are the Jacoby polynomials, orthogonal on the segment $[-1,1]$ with the weight $\rho(t)$. We determine unknown coefficients by the least square method from the system of linear algebraic equations (SLAE)

$$\sum_{k=1}^{n} \alpha_k (K\varphi_k, K\varphi_j) = (f, K\varphi_j), \quad j = \overline{1,n}, \tag{4}$$