Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 44, No. 11, pp.31–38, 2000 *UDC 517.968*

ON NONLINEAR INTEGRAL EQUATION OF FIRST KIND

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The article is devoted to the study of a nonlinear integral equation of first kind, As is well-known, many inverse problems for differential equations can be reduced to integral equations of first kind (see, e. g., [1], [2]). In the investigation of inverse coefficient problems for nonlinear differential equations there arise the integral equations of first kind with respect to the unknown function whose argument is a given function of two variables (see [3]). The linear integral equations of this type were studied in [4]. In this article we prove the existence and uniqueness of a solution of a nonlinear integral equation of first kind, in which the argument of the unknown function is a given function of two variables.

Let us consider the nonlinear integral equation

$$\int_0^l K(x,t,\varphi(u(x,t)))\,\mathrm{d}x = f(t), \quad t \in [0,T], \tag{1}$$

where the functions $K(x,t,s)$, $u(x,t)$ are given and $\varphi(z)$ is unknown.

Suppose that

$$u,\, u_x,\, u_t,\, u_{xt},\, u_{xx} \in C([0,l]\times[0,T]), \tag{2}$$

$$u(x,0)=0, \;\; u_{xt}(x,0)>0, \quad x\in[0,l], \tag{3}$$

$$u_x(x,t)>0, \;\; u_t(x,t)\ge \text{const}>0, \quad x\in[0,l], \;\; t\in(0,T], \tag{4}$$

and the function $u_{xx}(x,t)[u_x(x,t)]^{-1}$ can be extended so that

$$u_{xx}(x,t)[u_x(x,t)]^{-1} \in C([0,l]\times[0,T]). \tag{5}$$

Further, we suppose that

$$f \in C^1([0,T]), \tag{6}$$

and the function $K(x,t,s)$ satisfies the following conditions for any $x\in[0,l]$, $t\in[0,T]$, $s_1,s_2\in\mathbf{R}$:

$$K,\, K_x,\, K_t \in C([0,l]\times[0,T]\times\mathbf{R}), \tag{7}$$

$$|K(x,t,s_1)-K(x,t,s_2)| \le k_1(x,t)|s_1-s_2|, \tag{8}$$

$$|K_x(x,t,s_1)-K_x(x,t,s_2)| \le k_2(x,t)|s_1-s_2|, \tag{9}$$

$$|K_t(x,t,s_1)-K_t(x,t,s_2)| \le k_3(x,t)|s_1-s_2|, \tag{10}$$

where $k_i \in C([0,l]\times[0,T])$ $(i=1,2,3)$. In addition, the equation

$$\int_0^l K(x,0,s)\,\mathrm{d}x = f(0) \tag{11}$$

Partially supported by the Italian Ministry of Scientific and Technological Research and the Russian Foundation for Basic Research (project 96-15-96181).