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Classification of Complex Simply Connected Homogeneous Spaces of Dimension not Greater than 2

V. V. Gorbatsevich[1*]

[1] *Russian State Technological University, ul. Orshanskaya 3, Moscow, 121552 Russia*

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Abstract—The paper is devoted to the classification of finite-dimensional complex Lie algebras of analytic vector fields on the complex plane and the corresponding actions of Lie groups on complex two-dimensional manifolds. These Lie algebras were specified by Sophus Lie. He specified vector fields which form bases of the Lie algebras. However the structure of the Lie algebras was not clarified, and isomorphic Lie algebras among listed were not established. Thus, the classification was far from complete, and the situation has not been essentially changed until now. This paper is devoted to the completion of the above mentioned classification. We consider the part of this classification which concerns transitive actions of Lie groups.

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INTRODUCTION

The paper is devoted to a classical theme, namely to the classification of finite-dimensional complex Lie algebras of analytic vector fields on the complex plane and the corresponding actions of Lie groups on complex two-dimensional manifolds. These Lie algebras were specified by Sophus Lie [1]. But the form in which they were presented was far from perfect. He specified only vector fields which form bases of the Lie algebras. The structures of the Lie algebras (even their Levi decompositions, but it should be noted that the Levi decomposition was not known at that time) were not clarified. Also, non-isomorphic Lie algebras among listed were not singled out. Thus, the classification was far from complete, and the situation has not been essentially changed until now. This paper is devoted to the completion of the above mentioned classification. We consider here the part of this classification which concerns transitive actions of Lie groups. The case of intransitive actions is considerably simpler, it will be considered in another author's paper devoted to Lie subalgebras of Lie algebras and vector fields and their deformations.

In this paper, we consider only complex manifolds and Lie groups and only complex Lie algebras. Actions of Lie groups and vector fields are assumed to be complex analytic. The real case is studied in perfect analogy, it will not be considered here.

A homogeneous space of a Lie group G is represented in the form $M = G/H$, where G is a Lie group (supposed, in this paper, to be connected and, if necessary, simply connected) and H is the stationary subgroup being a Lie subgroup in G. If M is a complex manifold, then G and H are supposed to be complex. If in addition M is simply connected, the subgroup H is supposed to be connected. Therefore, from the study of a pair (G, H) one can pass to the study of the corresponding pair of Lie algebras (g, h). However, a pair of Lie algebras (g, h) is not always globalized, i.e., the subgroup H will not always be closed in G, even if G is simply connected, therefore, H will not always be the stationary subgroup of the homogeneous space G/H. For example, the Lie subalgebra h which corresponds to an irrational winding of the two-dimensional torus (e.g., of a maximal torus in the Lie group $G = SU(2) \times SU(2)$) is not globalized. However, as it has been proved in [2], in the case $\mathrm{codim}_G H \le 4$ (the codimension is taken in the real sense), any pair (g, h) is globalized. Since, in this paper, we consider only homogeneous

[*] E-mail: vgorvich@yandex.ru.