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A SUFFICIENT CONDITION FOR NILPOTENCE OF THE COMMUTANT OF THE LIE ALGEBRA

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In the present exposition we shall follow the monograph [1], where one can find all not explained notions, with the unique difference related to the author's habit to omit brackets in the case of their left-normed disposition, i. e., $xyz = ((xy)z)$.

The characteristic of the basic field within the whole article is assumed to be zero. As usual, we denote by $\mathbf{A}$ the variety of the Abelian Lie algebras, $\mathbf{N}_c$ stands for the variety of nilpotent algebras with their nilpotence degree not exceeding c. Then $\mathbf{N}_c\mathbf{A}$ is the variety of the Lie algebras, whose commutants are nilpotent with a nilpotence degree not exceeding c. The latter variety is defined by the identity

$$(x_0y_0)(x_1y_1)(x_2y_2)\cdots(x_cy_c) = 0. \tag{1}$$

Let us consider two identities which for $m \geq c$ are consequences of identity (1):

$$(x_0y_0)(xy)^m = 0, \tag{2}$$

$$\sum_{p\in S_m,\, q\in S_m} (-1)^p(-1)^q(x_0y_0)(x_{p(1)}y_{q(1)})(x_{p(2)}y_{q(2)})\cdots(x_{p(m)}y_{q(m)}) = 0. \tag{3}$$

In the present article we investigate the question: Whether identity (1) follows from identities (2) and (3). Let us note that each separate identity either (2), or (3), does not imply nilpotence of the commutant even for small values of m, additional condition of a decidability of the degree 3, and a strong condition upon the variety growth.

For example, in the complete list of decidable varieties of almost polynomial growth $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$, and $\mathbf{V}_4$ (see section 5 of the survey [2]) a nilpotent commutant belongs only to the algebras of the variety $\mathbf{V}_1 = \mathbf{N}_c\mathbf{A}$. At the same time, in the variety $\mathbf{V}_2$ identity (2) takes place for $m \geq 3$, and in the varieties $\mathbf{V}_3$ and $\mathbf{V}_4$ identities (3) are fulfilled for $m \geq 5$.

One can readily see that these two identities taken together are weaker than the Engel condition of index m for a commutant. The main result of the present article is the proof of nilpotence of a commutant under the condition of fulfillment of the given identities and exponential growth of the variety (see § 2). The question about possibility to remove the condition upon the variety growth remains open and, perhaps, very complicated. In Section 1 we expose the proof of a combinatorial result, which is a non-associative analog of the famous Shirshov lemma. By its ideology, the present article is close to the author's paper [3].

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