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On Solvability of Nonlocal Problem for Loaded Parabolic-Hyperbolic Equation

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Abstract—We study unique solvability of a nonlocal problem for equations of mixed type in a finite domain. This equation contains the partial fractional Riemann–Liouville derivative. The boundary condition of the problem contains a linear combination of operators of fractional differentiation in the sense of Riemann–Liouville of values of function derivative on the degeneration line and generalized operators of fractional integro-differentiation in the sense of M. Saigo. The uniqueness theorem of the problem is proved by a modified Tricomi method. The existence of solutions is equivalently reduced to the solvability of Fredholm integral equation of the second kind.

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1. Definition of the problem. Let us consider the equation

$$\begin{aligned} u_{xx} - D^{\alpha}_{0+,y} u = 0 \ \ (y > 0,\ 0 < \alpha < 1), \\ y^{2m} u_{xx} + y\, u_{yy} + \lambda u_y = 0 \ \ (y < 0), \end{aligned} \tag{1}$$

where $D^{\alpha}_{0+,y}$ is the partial fractional Riemann–Liouville derivative of order α $(0 < \alpha < 1)$ of the function $u(x, y)$ with respect to the second variable ([1], P. 341):

$$\left(D^{\alpha}_{0+,y} u\right)(x, y) = \left(\frac{\partial}{\partial y}\right) \frac{1}{\Gamma(1-\alpha)} \int_0^y \frac{u(x,t)\, dt}{(y-t)^{\alpha}} \quad (0 < \alpha < 1,\ y > 0), \tag{2}$$

m is a natural number, $\lambda = \text{const}$, $\frac{1-2m}{2} \leqslant \lambda < 1$, in the finite domain D bounded by segments AA_0, BB_0, A_0B_0 of right lines $x = 0$, $x = 1$, $y = 1$, respectively, and characteristics of Eq. (1) with $y < 0$

$$AC : x - \frac{2}{2m+1} (-y)^{\frac{2m+1}{2}} = 0, \quad BC : x + \frac{2}{2m+1} (-y)^{\frac{2m+1}{2}} = 1.$$

Let $D_1 = D \cap (y > 0)$, $D_2 = D \cap (y < 0)$, $I \equiv AB$ be a unit interval $0 < x < 1$ of the right line $y = 0$.

Problem. Find a solution $u(x, y)$ to Eq. (1) in the domain D, which satisfies the boundary conditions

$$u(0, y) = \varphi_1(y), \quad u(1, y) = \varphi_2(y), \quad 0 \leqslant y \leqslant 1, \tag{3}$$

$$\begin{aligned} a(x)\big(I^{\alpha_1,\beta_1,\eta_1}_{0+} w(t) u[\Theta_0(t)]\big)(x) + b(x)\big(I^{\alpha_2,\beta_2,\eta_2}_{1-} \delta(t) u[\Theta_1(t)]\big)(x) \\ + c(x)\big(D^{2\beta-1}_{0x} \lim_{y\to 0-0} (-y)^{\lambda} u_y(t, y)\big)(x) + d(x)\big(D^{2\beta-1}_{x1} \lim_{y\to 0-0} (-y)^{\lambda} u_y(t, y)\big)(x) \\ + g(x) \lim_{y\to 0-0} (-y)^{\lambda} u_y(x, y) + h(x) u(x, 0) = \gamma(x) \ \forall\, x \in I, \end{aligned} \tag{4}$$

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