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Necessary and Sufficient Conditions of Polynomial Growth of Varieties of Leibniz–Poisson Algebras

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Abstract—Leibniz–Poisson algebras are generalizations of Poisson algebras. We give equivalent conditions of polynomial growth of a variety of Leibniz–Poisson algebras over a field of characteristic zero. We find all varieties of Leibniz–Poisson algebras with almost polynomial growth belonging to a certain class of varieties.

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The paper is devoted to numerical characteristics of varieties of Leibniz–Poisson algebras over a field of characteristic zero. We give necessary and sufficient conditions of polynomial growth for such varieties.

Unless otherwise stipulated, throughout the paper we assume that the basic field K has characteristic zero. We define a Leibniz–Poisson algebra as follows. An algebra $A = A(+, \cdot, \{,\}, K)$ over a field K is called a Leibniz–Poisson algebra if $A(+, \cdot, K)$ is an associative commutative algebra with unity, $A(+, \{,\}, K)$ is a Leibniz algebra with multiplication operation $\{,\}$, and the relations

$$\{a \cdot b, c\} = a \cdot \{b, c\} + \{a, c\} \cdot b,$$
$$\{c, a \cdot b\} = a \cdot \{c, b\} + \{c, a\} \cdot b$$

hold for any $a, b, c \in A$. A Leibniz algebra $A(+, \{,\}, K)$ over a field K is defined by the identity

$$\{\{x, y\}, z\} = \{\{x, z\}, y\} + \{x, \{y, z\}\},$$

i.e., the right multiplication by an element of the algebra is a derivation.

Note that if the identity $\{x, x\} = 0$ holds in a Leibniz algebra, this algebra is a Lie algebra. Therefore, if this identity holds in a Leibniz–Poisson algebra, this algebra is a Poisson algebra. Poisson algebras arise in various divisions of Algebra, Differential Geometry, Topology, modern Theoretical Physics (see, e.g., [1]), and others.

We will omit the brackets $\{,\}$ in the cases when they are left-normed arranged, i.e.,

$$\{\{\{x_1, x_2\}, x_3\}, \dots, x_n\} = \{x_1, x_2, \dots, x_n\}.$$

Likewise, by $\{y, x^n\}$ we will mean the left-normed monomial $\{y, x, dotsc, x\}$, where the variable x occurs exactly n times.

Let $L(X)$ be a free Leibniz algebra where $X = \{x_1, x_2, \dots\}$ is a countable set of free generators, and let $F(X)$ be a free Leibniz–Poisson algebra. Denote by P_n the subspace in $F(X)$ consisting of multilinear elements of degree n in variables $x_1, \dots, x_n$.

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