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Automaton Transformations and Monadic Theories of Infinite Sequences

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Abstract—In this paper we prove that the set of degrees of asynchronously automaton transformations of infinite sequences with a solvable monadic theory forms an initial segment in the set of degrees of asynchronously automaton transformations. We prove a solvability criterion for a monadic theory of a complete sequence.

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In [1] one defines an equivalence relation for infinite sequences over finite alphabets with the help of Mealy automata, and defines a partial order on equivalence classes. In [2] these definitions are generalized for the case of asynchronous automata.

Definition 1 ([3], pp. 14, 35)**.** A finite Mealy automaton (a finite asynchronous automaton) is a collection $T = (S, \Sigma, \Sigma', \delta, \omega)$, where S, Σ, and Σ' are finite sets of states, input, and output symbols, correspondingly; $\delta : S \times \Sigma \longrightarrow S$ is the next-state function; $\omega : S \times \Sigma \longrightarrow \Sigma'$ (respectively, $\omega : S \times \Sigma \longrightarrow (\Sigma')^*$) is the output function. If an initial state s_0 is selected, then the automat (T, s_0) is called initial.

Definition 2 ([1, 2])**.** Let x and y be infinite sequences over finite alphabets. The sequence y is automatly reducible (asynchronously automatly reducible) to the sequence x, if there exists a finite initial Mealy automaton (a finite initial asynchronous automaton) (T, s_0) such that $\omega_T(s_0, x) = Ay$, where the block A defines some finite delay (respectively, $\omega_T(s_0, x) = y$).

This reducibility relation induces an equivalence relation on the set of infinite sequences. The equivalence class of a sequence x is called the degree of automaton transformations (correspondingly, the degree of asynchronously automaton transformations) and denoted by $[x]$ (correspondingly, by $[x]^*$) [1, 2].

V. R. Bairasheva [4] has proved that the solvability of the monadic theory of a sequence x implies the solvability of the monadic theory of a sequence y, if $[y] \leq [x]$. Let us generalize this result for the case of the asynchronously automaton reducibility.

One can formulate a solvability criterion for the monadic theory of an infinite sequence in terms of the automata theory. Namely, the monadic theory of a sequence x is solvable if and only if there exists an algorithm which for any Büchi automaton (or any deterministic Muller automaton) can find out whether this automaton accepts the sequence x or not (see, for example, corollary 3.1.4 in [5]).

One can find definitions of the monadic theory of sequences and Büchi and Muller automata, for example, in [5]. We denote the monadic theory of a sequence x by $MT\langle \mathbb{N}, <, x\rangle$, we do the set of sequences accepted by a Büchi or Muller automaton S by L_S.

Theorem 1. *If* $[y]^* \leq^* [x]^*$ *and* $MT\langle \mathbb{N}, <, x\rangle$ *is solvable, then so is* $MT\langle \mathbb{N}, <, y\rangle$.

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