

Stability of Solutions to Differential Equations With Several Variable Delays. I

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Abstract—We consider a class of scalar linear differential equations with several variable delays and constant coefficients. A family of equations of the class is defined by coefficients and maximum admissible values of delays. We obtain conditions that are necessary and sufficient for the stability of solutions to all equations of the family. It is ascertained that the conditions are determined entirely by properties of the solution to the initial problem for an autonomous equation that belongs to the family. Some alternatives of required conditions are obtained in the form of estimates for solutions to autonomous equations in a finite interval.

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This paper presents the first part of the investigation of stability of solutions to scalar differential equations with several variable delays. The finding of efficient criteria of stability defines the necessity of investigation of other asymptotic properties of solution, in particular, the fixed sign property and the boundedness. Since recent years the number of papers, which are dedicated to these questions, increases, we consider that with the results exposition it is necessary to aim the best clarity of definition of their place in the general image of knowledge about the subject. This aim essentially defines both the set of results included in the present paper and the structure of reasoning.

The main content of the paper is the proof of a series of assertions, which establish the used method of obtaining efficient stability criteria. The first Section presents the short review of the main papers, in which one has obtained the efficient stability criteria for solutions to equations, which belong to the investigated class. The second Section is dedicated to the problem definition. We introduce the notion of *stability of equations family*, which aids to clarify the understanding of the *exactness* of the stability domain and the significance of the obtained results. In the third Section we adduce the main results of the paper: We prove the assertions, which connect the properties of solutions to nonautonomous equations from certain family with properties of solution to the autonomous equation corresponding to this family; we obtain criteria of stability for all equations of the family.

1. THE HISTORY OF QUESTION

Even in the scalar case, one can obtain necessary and sufficient conditions of stability for linear equations with delayed argument expressed in terms of initial parameters for chosen classes of equations only: for autonomous equations ([1], Chap. 10), for some equations with periodic coefficients [2, 3], and for some equations with delay of a special form [4], close to ordinary differential equations (ODE) by properties. Therefore for equations with an arbitrary variable delay one find *sufficient* conditions (*criteria*) of stability. These criterions may have different strength. Naturally, one should prefer the ones, for which one can show the substantiality of all conditions and the unimprovability of boundaries of stability domains in the space of equation parameters. Apparently, the results of A. D. Myshkis about equations with one variable delay begin the history of such criteria.

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