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Convergence in the Integral Metric of the General Projective Method for Solving Singular Integral Equations of the First Kind with the Cauchy Kernel

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Abstract—We study a projective method for solving singular integral equations of the first kind with the Cauchy kernel. Depending on the index of the equation, we introduce pairs of weight spaces which represent a restriction of the space of summable functions. We prove the correctness of the stated problem. We obtain sufficient conditions for the convergence of the projective method in the integral metric.

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INTRODUCTION

In this paper we study the general projective method for solving singular integral equations (s. i. e.) of the first kind with the Cauchy kernel in the form

$$A\varphi \equiv \frac{1}{\pi}\int_{-1}^{+1}\frac{\varphi(\tau)}{\tau - t}d\tau + \frac{1}{\pi}\int_{-1}^{+1} h(t;\tau)\varphi(\tau)d\tau = f(t), \quad |t| < 1, \tag{0.1}$$

where $h(t;\tau)$ and $f(t)$ are known functions, $\varphi(\tau)$ is the desired function, and the singular integral

$$I\varphi = I(\varphi; t) = \frac{1}{\pi}\int_{-1}^{+1}\frac{\varphi(\tau)}{\tau - t}d\tau$$

is understood as the Cauchy–Lebesgue principal value.

Such equations are of great interest, because they occur in many theoretical and applied problems (see, e.g., [1, 2] and references therein). The main difficulty of solving Eq. (0.1) in the known functional spaces is connected with its incorrectness. The theory of such equations is developed sufficiently well ([2–4]), however, one can obtain a solution to Eq. (0.1) in closed form only in rare particular cases. Therefore equations in the form (0.1) are solved by various approximate methods. These methods require the theoretical substantiation and estimation of errors. In spite of a sufficiently large amount of works in this area (see, e.g., [1] and references therein) the question on the convergence of approximate methods for solving s. i. e. (0.1) in the widest class of functions (namely, in the space $L = L[-1, 1]$ of functions summable on the segment $[-1, 1]$) remains open. It is well-known (see, e.g., [3, 4]) that the index of Eq. (0.1) takes on three values: 1) $\varkappa = 1$, 2) $\varkappa = 0$, 3) $\varkappa = -1$. Depending on this index, we seek for a solution $\varphi(t)$ in the classes of functions which are 1) unbounded at both ends, 2) bounded at one end and unbounded at another one, 3) bounded at both ends. For these classes the function $\varphi(t)$ admits the representation $\varphi(t) = \rho(t)x(t)$, where $x(t)$ is a new desired function, and $\rho(t)$ is defined, correspondingly, as follows: 1) $\rho(t) = \frac{1}{\sqrt{1-t^2}}$, 2) $\rho(t) = \sqrt{\frac{1\pm t}{1\mp t}}$, 3) $\rho(t) = \sqrt{1-t^2}$.

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