

THE SOLVABILITY OF DEGENERATE SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

O.V. Bormotova, S.V. Gaidomak, and V.F. Chistyakov

1. Introduction

Consider the system of equations

$$\Lambda_{1,1}u := A \frac{\partial u}{\partial t} + B \frac{\partial u}{\partial x} + Cu = f, \quad (1.1)$$

where A , B , C are constant $n \times n$ -matrices, $u \equiv u(x, t)$, $f \equiv f(x, t)$ are the desired and given vector-functions, respectively, $(x, t) \in U = [0, x_0] \times [0, t_0] \subset \mathbf{R}^2$. Assume that

$$\det A = 0 \quad (1.2)$$

and the vector-function f is sufficiently smooth in the domain U .

Within the last ten years systems (1.1) which fulfill condition (1.2) attract more and more attention, because they arise in diverse application areas [1]–[3].

Below we widely use the methods elaborated during the investigation of linear systems of ordinary differential equations

$$\Lambda_1\chi := A \frac{d\chi}{dt} + B\chi = \varphi, \quad t \in [0, t_0], \quad (1.3)$$

where $\chi \equiv \chi(t)$, $\varphi \equiv \varphi(t)$ are the desired and given vector-functions, respectively, and condition (1.2) holds.

2. Existence theorems

Let us study some properties of system (1.1), assuming that the matrix $C = 0$, and the pencil of matrices $\lambda A + B$ is regular. The symbol λ stands in this paper for a numeric parameter or an operator.

Definition 2.1. A pencil of matrices $\lambda A + B$ is *regular*, if $\det(\lambda A + B) \neq 0$, where λ is a numeric parameter.

Under our assumptions, we have the system of equations

$$A \frac{\partial u}{\partial t} + B \frac{\partial u}{\partial x} = f. \quad (2.1)$$

In order to investigate it, we need the following lemma.

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