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THE SOLVABILITY OF DEGENERATE SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

O.V. Bormotova, S.V. Gaidomak, and V.F. Chistyakov

1. Introduction

Consider the system of equations

$$\Lambda_{1,1}u := A\frac{\partial u}{\partial t} + B\frac{\partial u}{\partial x} + Cu = f, \tag{1.1}$$

where A, B, C are constant $n \times n$-matrices, $u \equiv u(x,t)$, $f \equiv f(x,t)$ are the desired and given vector-functions, respectively, $(x,t) \in U = [0,x_0] \times [0,t_0] \subset \mathbf{R}^2$. Assume that

$$\det\ A = 0 \tag{1.2}$$

and the vector-function f is sufficiently smooth in the domain U.

Within the last ten years systems (1.1) which fulfill condition (1.2) attract more and more attention, because they arise in diverse application areas [1]–[3].

Below we widely use the methods elaborated during the investigation of linear systems of ordinary differential equations

$$\Lambda_1\chi := A\frac{d\chi}{dt} + B\chi = \varphi, \quad t \in [0,t_0], \tag{1.3}$$

where $\chi \equiv \chi(t)$, $\varphi \equiv \varphi(t)$ are the desired and given vector-functions, respectively, and condition (1.2) holds.

2. Existence theorems

Let us study some properties of system (1.1), assuming that the matrix $C = 0$, and the pencil of matrices $\lambda A + B$ is regular. The symbol λ stands in this paper for a numeric parameter or an operator.

Definition 2.1. A pencil of matrices $\lambda A + B$ is *regular*, if $\det(\lambda A + B) \not\equiv 0$, where λ is a numeric parameter.

Under our assumptions, we have the system of equations

$$A\frac{\partial u}{\partial t} + B\frac{\partial u}{\partial x} = f. \tag{2.1}$$

In order to investigate it, we need the following lemma.