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Approximating Characteristic Equations for Autonomous Systems of Differential Equations with Aftereffect

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Abstract—In this paper we construct approximating polynomial characteristic equations for a linear autonomous system with aftereffect. The procedures for constructing approximating characteristic equations use analytic representations of resolvents of infinitesimal operators and the theory of characteristic determinants and perturbation determinants in a separable Hilbert space.

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We consider the following linear autonomous system of differential equations with aftereffect:

$$\frac{dx(t)}{dt} = \int_{-r}^{0} d\eta(\vartheta)x(t+\vartheta), \quad t \in \mathbb{R}^+ = (0, +\infty), \tag{1}$$

where $x : [-r, +\infty) \to \mathbb{R}^n$ and η is a matrix function with bounded variation on $[-r, 0]$, $\eta(0) = 0$.

In the functional state space $\mathbf{C}\big([-r, 0], \mathbb{R}^n\big)$ we associate system (1) with the equation

$$\frac{dx_t}{dt} = Ax_t, \ \ t \in \mathbb{R}^+,$$

where $(Ax)(\vartheta) = \frac{dx(\vartheta)}{d\vartheta}$, $\vartheta \in [-r, 0]$; $D(A) = \Big\{x(\cdot) : x(\cdot) \in \mathbf{C}^1\big([-r, 0],\ \mathbb{R}^n\big),\ x'(0) = \int\limits_{-r}^{0} d\eta(\vartheta)x(\vartheta)\Big\}$.

The spectrum of the operator A consists of eigenvalues which are roots of the transcendental characteristic equation ([1], Chap. 6)

$$D(\lambda) = \det\left(\lambda I_n - \int_{-r}^{0} d\eta(\vartheta)\exp(\lambda\vartheta)\right) = 0, \quad \lambda \in \mathbb{C}. \tag{2}$$

Here I_n is the identity matrix of the dimension $n \times n$, $\mathbb{C}$ is the set of complex numbers, and D is an entire function of the exponential type [2].

One has to construct approximating polynomial characteristic equations for systems of differential equations with aftereffect. To this end one uses the connection between the spectrum of the infinitesimal operator A and its resolvent $R_0 = (\lambda_0 I_n - A)^{-1}$, where λ_0 is some regular point of the operator A.

The obtained results are applicable in calculating roots of characteristic equations, in problems of the exponential stability of autonomous systems of differential equations with aftereffect [3, 4], in studying the stability of a viscoelastic rod [5, 6], and in problems of the optimal stabilization of systems with aftereffect [7–9].

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