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The Set of Quantum States and its Averaged Dynamic Transformations

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Abstract—In this paper we consider the set of quantum states and passages to the limit for sequences of quantum dynamic semigroups in the mentioned set. We study the structure of the set of extreme points of the quantum state set and represent an arbitrary state as an integral over the set of one-dimensional orthogonal projectors; the obtained representation is similar to the spectral decomposition of a normal state. We apply the obtained results to the analysis of sequences of quantum dynamic semigroups which occur in the regularization of a degenerate Hamiltonian.

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Introduction. The degeneracy of the Hamiltonian on a subset of the phase or coordinate space leads to an ill-posed statement of the Cauchy problem for the evolution equation both in the classical [1] and quantum [2] mechanics. A dynamic transformation of the state space of a system with a degenerate Hamiltonian is understood as the limit of a sequence of regularized dynamic semigroups defined by regularized Hamiltonians [3]. In view of limit passages in the space of quantum states one studies the limiting dynamics of a quantum system not in a subset of normal states but in a set of states of a general type ([4, 5]). The latter states, unlike the normal ones, cannot be represented by kernel or bounded linear operators in a Hilbert space of a quantum system [6–8]. Therefore, in order to study these limit passages one needs to apply methods developed for analyzing kernel operators to studying states of a general type. Namely, one needs to determine the commutativity relation between a state and an operator or between two states, to describe the structure of the set of extreme points of the state set, and to study the representability of an arbitrary state as an integral over the set of one-dimensional orthogonal projectors in the spirit of the spectral theorem for normal states.

Quantum states, weights, and measures on projectors of the algebra $B(H)$ are studied in papers by J. von Neumann, F. Murray, A. Gleason, I. Segal, J. Mackey, and many other authors (see [9] and references therein). The Gleason theorem establishes a connection between completely additive measures on projectors and normal states. Theorem 27.16 in [9] is a generalization of the Gleason theorem for the case of unbounded measures. It states that a measure defined on projectors of the algebra $B(H)$ determines a weight on $B(H)$. The goal of this paper is to study the representation of states which are not normal by means of measures on the set of one-dimensional orthogonal projectors.

One can represent a normal state by pairwise orthogonal one-dimensional projectors of a certain orthogonal decomposition of the unit operator (see the Gleason theorem and the spectral theorem). One can also represent it with the help of an overfilled system of one-dimensional orthogonal projectors (for example, by the Glauber states [10]). In the first case the decomposition of the state is called *orthogonal*, and in the second case it is said to be *nonorthogonal*. In this paper for an arbitrary quantum state we obtain a nonorthogonal decomposition in the form of the covering of the set of vector states. We study a special class of states decomposable into the Pettis integral over the family of pairwise orthogonal one-dimensional projectors.

We apply the obtained results to the description of the dynamics of quantum systems with degenerate Hamiltonians and for their regularization.

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