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The Asymptotic Stability of a Nonstationary System with Delay

B. G. Grebenshchikov[1*]

[1] *Ural Federal University, ul. Mira 19, Ekaterinburg, 620002 Russia*[1]

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Abstract—In this paper we study the exponential stability of a linear system of differential equations with a constant delay such that the right-hand side of one of its subsystems contains an exponential multiplier. We establish sufficient conditions for the existence of an asymptotically periodic solution to the inhomogeneous system.

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1. STABILITY OF NONSTATIONARY HOMOGENEOUS SYSTEMS

Consider the following linear homogeneous system with a constant delay:

$$\begin{gathered} dx(t)/dt = A_1(t)x(t) + A_2(t)x(t-\sigma) + B_1(t)y(t) + B_2(t)y(t-\sigma), \\ dy(t)/dt = \theta_0 e^t[A_3x(t) + A_4x(t-\sigma) + B_3y(t) + B_4y(t-\sigma)], \\ t \geq 0, \ \ \theta_0 > 0, \ \ \sigma = \text{const}, \ \ \sigma > 0. \end{gathered} \tag{1.1}$$

Here $A_i(t)$ and $B_i(t)$ $(i = 1, 2)$ are periodic (with a period $0 < T \leq \sigma$) continuously differentiable $m \times m$-matrices, A_k and B_k $(k = 3, 4)$ are constant $m \times m$-matrices, while $x(t)$ and $y(t)$ are m-dimensional vector functions with respect to (the argument) t.

We define the norm of a vector $w = \{w_j\}^{(\top)}$ (here w_j are components of the vector w, and $(^\top)$ is the transposition sign) by the equality $\|w\| = \sum\limits_{j=1}^{m} |w_j|$; we do the norm of a matrix $D = \{d_{ij}\}$ $(i, j = 1, \ldots, m)$ in accordance with the norm of a vector ([1], P. 12), namely, $\|D\| = \max\limits_j \sum\limits_i |d_{ij}|$.

Earlier we studied properties of a system with constant matrices A_i^0 and B_i^0 $(i = 1, 2)$ in papers [2–4]. Note that one can obtain system (1.1) by replacing the time variable $t = \ln(\frac{\theta}{\theta_0})$ in the following system with a linear delay:

$$\begin{gathered} d\widehat{x}(\theta)/d\theta = \frac{1}{\theta}[\widehat{A}_1(\theta)\widehat{x}(\theta) + \widehat{A}_2(\theta)\widehat{x}(\mu\theta) + \widehat{B}_1(\theta)\widehat{y}(t) + \widehat{B}_2(\theta)\widehat{y}(\mu\theta)], \\ d\widehat{y}(\theta)/d\theta = A_3\widehat{x}(\theta) + A_4\widehat{x}(\mu\theta) + B_3\widehat{y}(\theta) + B_4y(\mu\theta), \\ \theta \geq \theta_0 > 0, \ \ \sigma = -ln(\mu), \ \ 0 < \mu < 1. \end{gathered}$$

On an initial set $[\mu\theta_0, \theta_0]$ we define an initial vector function $\widehat{\phi}(\theta)$. We can solve this system by successive integration on segments $h_n(\theta) = \left(\frac{\theta_0}{\mu^n}, \frac{\theta_0}{\mu^{n+1}}\right]$. This method does not allow one to establish asymptotic properties of the system under consideration.

[1] E-mail: ablozhnikov@yandex.ru
[*] E-mail: ablozhnikov@yandex.ru.