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On the Problem of Classification of Regular 4-Webs Formed by Pencils of Spheres

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Abstract—Shelehov's theorem on bondary curves of a regular curvilinear 3-web is generalized to the case of an arbitrary regular codimension 1 $(n+1)$-web; an example is given of a regular 4-web formed by pencils of spheres in the three-dimensional conformal space (W. Blaschke's problem); it is proved that a spherical 4-web of the basic type cannot be regular.

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INTRODUCTION

In [1], the two "conformal" problems have been formulated: Find all regular (parallelizable) 3-webs formed by pencils of circles and give examples of 4-webs formed by pencils of spheres in the three-dimensional conformal space. Recall that a 3-web is called *parallelizable* or *regular* if it is equivalent to a web formed by families of parallel planes.

Several authors were involved in the study of the first problem, however, in spite of its seemingly simple formulation, the problem resists solution. The matter is that all the authors, following the way suggested by Blaschke, calculated the curvature of a web and equated it to zero. This way led to cumbersome calculations which no one could bring to an end. In [2], one of the authors of this paper suggested another way based on the simple, in outward appearance, fact: It occurs that the smooth part of the boundary curve of any regular curvilinear 3-web belongs to the web. With the use of this fact, in [2], it has been strictly proved that there are no regular circle 3-webs except for the known seven classes listed in [3].

In this paper, we show that the problem of classification of regular 4-webs formed by pencils of spheres in the three-dimensional conformal space (briefly, spherical 4-webs) can efficiently be solved with the use of methods and results of [2].

First, we generalize the theorem on the boundaries of regular curvilinear 3-webs to the case of regular codimension 1 $(n+1)$-webs.

By definition ([4], P. 4), an $(n+1)$-web W on an n-dimensional manifold X is formed by $n+1$ foliations of codimension 1 in general position, which means that, through any point M of the domain of definition, there pass exactly $n+1$ leaves of the web, one from each of the foliations, and any n of these leaves are in general position, i.e., their tangent planes (at the point M) have zero intersection. In other words, the above mentioned tangent planes form a coframe at M.

Let the foliations λ_α, $\alpha = 1, 2, \ldots, n+1$, of an $(n+1)$-web W are given in local coordinates in a domain U by equations

$$F_\alpha(x^1, x^2, \ldots, x^n) = c_\alpha = \mathrm{const}\,. \qquad (0.1)$$

Then the condition "in general position" is equivalent to the fact that all the nth order determinants of the $n \times (n+1)$-matrix $\mathcal{G}$ formed by the gradients of the functions F_α are nonzero at each point of the domain of definition of the web.

The points of the domain U at which the rank of some of these determinants is less than n will be called, following [2], *boundary* points. These points do not belong to the domain of definition of the

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