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GEODESIC MAPS OF EINSTEIN SPACES

J. Mikeš and V.A. Kiosak

1. Introduction

A diffeomorphism f of a Riemannian space V_n onto a Riemannian space $\overline{V}_n$ is called *a geodesic map* if f maps any geodesic line of V_n onto a geodesic line of $\overline{V}_n$. Many authors studied these maps (see for instance [1]–[3]). For the list of recent papers on geodesic maps we refer the reader to [4].

Note that we do not impose any restriction on the metric signature of Riemannian spaces (cf. [2], [3]). We study Riemannian spaces locally and assume that all functions are sufficiently smooth.

The Einstein spaces which are characterized by

$$R_{ij} = \frac{R}{n} g_{ij},$$

where R_{ij} is the Ricci tensor, R is the scalar curvature, and g_{ij} is the metric tensor, are of great importance both in Riemannian geometry, and in its applications [1], [2],.... Many geometers study geodesic maps of Einstein spaces (see, e. g., [2], [4]–[13]).

Let us recall the final result by A.Z. Petrov and V.I.Golikov [2] on geodesic maps of four-dimensional Einstein spaces: *Let V_4 be a four-dimensional Einstein space V_4 with Minkowski signature. If the curvature of V_4 is nonconstant, then V_4 does not admit any nontrivial geodesic map onto a Riemannian space $\overline{V}_4$ with Minkowski signature.* We prove a theorem which generalizes this result.

A.Z. Petrov extended methods of studying geodesic maps of four-dimensional Einstein spaces with Minkowski signature to the Einstein spaces of higher dimensions $n > 4$, and conjectured that *the Einstein spaces V_n $(n > 4)$ with Minkowski signature which are distinct from the spaces of constant curvature, do not admit nontrivial geodesic maps onto the Einstein spaces with the same signature.* ([2], pp. 355, 461). In the present paper we give an example which disproves this statement.

It turned out that the Einstein spaces admitting geodesic maps, are necessarily the spaces $V_n(B)$ [4], [11]–[13], which generalize spaces $V(K)$ introduced in [14]. Therefore, first we present new results in the theory of geodesic maps of spaces $V_n(B)$, and then use them for studying geodesic maps of Einstein spaces.

2. Riemannian spaces $V_n(B)$

Let us denote by $V_n(B)$ ([4], [11], [13], [15]) a Riemannian space V_n admitting a nontrivial geodesic map such that

$$\begin{aligned} &\text{(a)}\ \ a_{ij,k} = \lambda_i g_{jk} + \lambda_j g_{ik}, \\ &\text{(b)}\ \ \lambda_{i,j} = \mu g_{ij} + B a_{ij}, \end{aligned} \tag{1}$$

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