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OPTIMIZATION OF HYPERBOLIC SYSTEMS WITH INTEGRAL CONSTRAINTS ON BOUNDARY CONTROLS

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Introduction

In this paper, we consider the problems of optimal control of initial boundary conditions of hyperbolic systems of the first order in the class of smooth controls. The systems under consideration can be obtained by the reduction of the classical hyperbolic equation of the second order, the systems of the Goursat–Darbou type and the canonical hyperbolic systems of the first order with the orthogonal systems of characteristics. In [1], the necessary optimality conditions for smooth boundary and starting controls satisfying the pointwise or integral constraints were obtained. In this paper, we propose one numerical method for solving the problems of optimization of the boundary and starting controls under integral constraints on control actions. We investigate two applied problems: the problem of optimal control of population with the given age distribution and the inverse problem of gravity wave profile restoration under the additional constraint which follows from the law of conservation of mass.

1. Problem definition and the necessary optimality condition

In the rectangle $P = S \times T$, $S = [s_0, s_k]$, $T = [t_0, t_k]$, we consider the system of semilinear hyperbolic equations

$$\frac{\partial x}{\partial t} + A(s,t)\frac{\partial x}{\partial s} = f(x,s,t), \quad s \in S, \quad t \in T. \tag{1}$$

Here $x = x(s,t)$ is n-dimensional vector-function. We assume that system (1) is written in the invariant form, i. e., the matrix $A(s,t)$ is diagonal (see [2], pp. 25–28).

In addition, we assume that the diagonal elements $a_i = a_i(s,t)$, $i = 1,2,\dots,n$, of the matrix of coefficients are of constant signs in P:

$$a_i(s,t) > 0, \ \ i = 1,2,\dots,m_1;$$
$$a_i(s,t) = 0, \ \ i = m_1+1, m_1+2,\dots,m_2;$$
$$a_i(s,t) < 0, \ \ i = m_2+1, m_2+2,\dots,n.$$

We compose two diagonal sumatrices: $A^+(s,t)$ of dimension $m_1 \times m_1$ and $A^-(s,t)$ of dimension $(n-m_2)\times(n-m_2)$ using the positive and negative diagonal elements of the matrix A, respectively.

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