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ENLARGED RECURRENT MANIFOLDS

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1. Introduction

Recurrent manifolds were first introduced and proved to exist by H.S. Ruse (see [1], [2]). A recurrent manifold is a non-flat n-dimensional analytic Riemannian manifold (M, g) whose curvature tensor satisfies the equality

$$\nabla_r R_{lkji} = A_r \cdot R_{lkji} \tag{1.1}$$

for a nonzero vector A_r, where ∇ is the covariant derivative with respect to g.

In [3] a second order recurrent space (2-recurrent space) was introduced. This is a Riemannian manifold whose curvature tensor satisfies the condition

$$\nabla_s \nabla_r R_{lkji} = C_{sr} R_{lkji}$$

for a tensor C_{sr}. V.R. Kaĭgorodov (see [4]–[6], also the survey [7]) developed a new method which made it possible to investigate manifolds satisfying the condition

$$\nabla_{m_s} \cdots \nabla_{m_2} \nabla_{m_1} R_{lkji} = \Omega_{m_1 m_2 \cdots m_s} R_{lkji} + \Omega_{m_2 \cdots m_1} \nabla_{m_1} R_{lkji} + \\ + \Omega_{m_3 \cdots m_s} \nabla_{m_2} \nabla_{m_1} R_{lkji} + \cdots + \Omega_{m_s} \nabla_{m_{s-1}} \cdots \nabla_{m_1} R_{lkji},$$

where $\Omega_{m_1 m_2 \cdots m_s} \neq 0$, $\Omega_{m_2 \cdots m_1}, \ldots, \Omega_{m_1}$ are tensors. He also studied applications of the recurrent manifolds in the general relativity theory.

Recently, works appeared, where the recurrent manifolds were also generalized, but along another lines (see [8]–[12]). The condition imposed there is

$$\nabla_r R_{lkji} = B_r R_{lkji} + A_l R_{rkji} + A_k R_{lrji} + A_j R_{lkri} + A_i R_{lkjr}. \tag{1.2}$$

Evidently, for $A_i = 0$ condition (1.2) can be reduced to (1.1). Another generalization is

$$\nabla_r R_{lkji} = 2A_r R_{lkji} + A_l R_{rkji} + A_k R_{lrji} + A_j R_{lkri} + A_i R_{lkjr}. \tag{1.3}$$

If the associated vector field A_r satisfies the condition

$$A_r R_{lkji} + A_j R_{lkir} + A_i R_{lkrj} = 0,$$

then (1.3) can also be reduced to (1.1).

In [13] we considered totally umbilic submanifolds of a manifold satisfying (1.2). It turns out that, in the general situation, such a submanifold also satisfies a condition of recurrent type, for example,

$$\nabla_r R_{lkji} = A_r[R_{lkji} + (\beta - \psi) G_{lkji}] + \frac{\beta}{2}(A_l G_{rkji} + A_k G_{lrji} + A_j G_{lkri} + A_i G_{lkjr}), \tag{1.4}$$