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BLACK HOLES AND QUASIBLACK HOLES: SOME HISTORY AND REMARKS

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Abstract

We give a short reference to the two Schwarzschild solutions and to what Petrov had to say about them. We comment on how the Schwarzschild vacuum solution describes a black hole. Then we compare the properties, differences and similarities between black holes and quasiblack holes. Black holes are well known. Quasiblack hole is a new concept. A quasiblack hole, either nonextremal or extremal, can be broadly defined as the limiting configuration of a body when its boundary approaches the body's own gravitational radius (the quasihorizon). Quasiblack holes are objects that are on the verge of being black holes but actually are distinct from them in many ways. We display some of their properties: there are infinite redshift whole regions; the curvature invariants remain perfectly regular everywhere, in the quasiblack hole limit; a freefalling observer finds in his own frame infinitely large tidal forces in the whole inner region, showing some form of degeneracy; outer and inner regions become mutually impenetrable and disjoint, although, in contrast to the usual black holes, this separation is of a dynamical nature, rather than purely causal; for external far away observers the spacetime is virtually indistinguishable from that of extremal black holes. We also discuss other important properties, such as the mass formula and the entropy, as compared to the corresponding properties of black holes.

Key words: Schwarzschild solution, Petrov, black holes, quasiblack holes.

1. Introduction

1.1. The Schwarzschild solution. Finding vacuum solutions of Einstein's equation

$$G_{ab} = 0, \tag{1}$$

where G_{ab} is the Einstein tensor, is an important branch of General Relativity and known to be a non-trivial task. On the other hand, finding solutions of the field equations with matter is a somewhat different setup. Given any metric, there is always one stressenergy tensor T_{ab} for which Einstein's equations (G = 1, c = 1)

$$G_{ab} = 8\pi T_{ab},\tag{2}$$

are trivially satisfied. Now, arbitrarily chosen metrics usually give rise to unphysical stress-tensors, corresponding to matter which is of no interest. Therefore, the task of finding non-vacuum solutions to the field equations is, in a certain way, twice as hard in comparison to solutions in vacuum, one has to choose physically relevant sources, and then solve for the gravitational field in the equations.

Schwarzschild, in 1916, in two strokes, initiated the field of exact solutions in General Relativity, both in vacuum [1] and in matter for an incompressible fluid [2]. These solutions are called the Schwarzschild solution and the interior Schwarzschild solution, respectively. The Schwarzschild solution [1] is perhaps the most well-known exact solution in General Relativity, and its line element can be written in appropriate spherical coordinates (t, r, θ, ϕ) as

$$ds^{2} = -\left(1 - \frac{2m}{r}\right) dt^{2} + \frac{dr^{2}}{\left(1 - \frac{2m}{r}\right)} + r^{2} \left(d\theta^{2} + \sin^{2}\theta \, d\phi^{2}\right).$$
(3)

Here m is the mass of the object, outside which there is vacuum. To interpret the solution as a whole vacuum solution, and the emergence of the notion of a black hole it took some time.

1.2. Petrov on the Schwarzschild solution. In a Petrov Symposium it is worth to spend some lines on what Petrov had to say on both Schwarzschild solutions. For this we refer to his book *Einstein Spaces*, published in Russian in 1961 and then translated into English in 1969 [3].

On page 141 of the book [3] one can read a rather remarkable phrase: "It is clear that Einstein, Hilbert, and their contemporaries had a rather primitive idea of what is meant by 'spacetime metric' and of its scope. They possessed only a few of the simplest examples (for example Schwarzschild's solution, the solution of Weyl and Levi-Civita with axial symmetry, and cosmological metrics). They did not realize what a powerful instrument they were forging."

Then there are several mentions, in passing, of the Schwarzschild solution. On page 179, it is stated that the Schwarzschild solution is a particular case of solutions included in T_1 , i.e., solutions with Segre characteristic (111), referring to his algebraic classification of 1954 of the Riemann and Weyl tensors [4], repeated in the book on page 99. On page 196, Kotler's solution is mentioned, stating it is a generalization of the Schwarzschild solution by including a cosmological term Λ . On page 360, in Chapter 9, Einstein's equations for a spherically symmetric vacuum are solved, and the Schwarzschild solution is finally displayed. On page 362, exercises on Schwarzschild and interior Schwarzschild are given, and the Landau and Lifshitz 1948 book *The Classical Theory of Fields* (and the English translation of 1959) is cited [5]. On page 386, the two Schwarzschild's papers of 1916 on the vacuum and the interior solutions are quoted in citations 37 and 37a, respectively.

There is an interesting contribution of Petrov to the field of exact solutions. In the paper Gravitational field geometry as the geometry of automorphisms [6], among many other solutions, Petrov finds a Type I (111) solution with metric $ds^2 = e^r \cos \sqrt{3}r(-dt^2 + d\phi^2) - 2 \sin \sqrt{3}r \, d\phi \, dt + dr^2 + e^{-2r} dz^2$. It is the only vacuum solution admitting a simply-transitive four-dimensional maximal group of motions. Bonnor [7] showed that it is the vacuum solution exterior to an infinite rotating dust, a particular case of the Lanczos-van Stockum solution. This is not a black hole, but has relations to the hoop conjecture, closed timelike curves, and so on.

1.3. Black holes. It is clear that the Schwarzschild solution (3) presents a problem, in the coordinates used, at r = 2m. For a long time r = 2m was a mysterious place. Only in the 1960s the ultimate interpretation was given and the problem was solved. The radius $r_{\rm h} = 2m$ defines the event horizon, a lightlike surface, of the solution. In its full form it represents a wormhole, with its two phases (the white hole and the black hole) connecting two asymptotically flat universes [8] (a work done under the supervision of Wheeler [9]). If, besides a mass m as in the Schwarzschild solution, one includes electrical charge q, the Reissner–Norström solution is obtained [10, 11] (for the interpretation of its full form see [12]). The inclusion of angular momentum J gives the Kerr solution [13], and the inclusion of the three parameters (m, J, q) is the Kerr-Newman family [14]. For a full account of the Kerr-Newman family within General Relativity see [15].

As predicted earlier by Oppenheimer and Snyder [16] black holes can form through gravitational collapse of a lump of matter. As the matter falls in, an event horizon develops from the center of the matter, and stays put, as a null surface, in the spherical symmetric case at $r_{\rm h} = 2m$, while the matter falls in towards a singularity. A posterior important result is that if the matter is made of perfect fluid (such as the Schwarzschild interior solution [2]) there is the Buchdahl limit [17] which states that when the bound-

ary of the fluid matter approaches quasistatically the value $\frac{9}{8}r_{\rm h}$, then the system ensues in an Oppenheimer-Snyder collapse, presumably into a black hole.

The possibility of existence of black holes came with Quasars in 1963. Salpeter [18] and Zel'dovich [19] were the first to advocate that a massive central black hole should be present in these objects in order to explain the huge amount of energy liberated by them. Lynden – Bell in 1969 then took a step forward and proposed that a central massive black hole should inhabit every galaxy [20], a prediction that has been essentially confirmed, almost every galaxy has a central black hole. Then with the discovery of pulsars in 1968 and the reality of neutron stars the possibility of small stellar mass black holes became obvious, confirmed in 1973 with the X-ray binary Cygnus X1 and then with other X-ray binary sources (see, e.g., [21]).

It is supposed that black holes can form in many ways. The traditional manner is the Oppenheimer-Snyder type collapse [16]. Nowadays, one also admits that black holes can form from the collision of particles, or have a cosmological primordial inbuilt origin (see, e.g., [21]). The Reissner-Nordström black hole may not be very useful astrophysically, although all black holes should have a tiny, fluctuating, charge. Notwithstanding, it might be important in particle physics, perhaps it is an elementary soliton of gravitation, as proposed by some supergravity ideas. Nowadays there is a profusion of theoretical black holes of all types, in all theories, with all charges, in all dimensions (see, e.g., [22]).

Classically, black holes are well understood from the outside: there is astrophysical evidence and theoretical consistency. Perhaps there will be phenomenological evidence in the near future from the collision of particles.

Quantically, black holes still pose problems. For the outside, these problems are related to the Hawking radiation and the Bekenstein-Hawking entropy. For the inside, the understanding of the inside of a black hole is one of the outstanding problems in gravitational theory, and it certainly is a quantum phenomenon. The horizon harbors a singularity. What is a singularity? The two quantum problems, the outside and the inside, are perhaps related. There are many approaches, some try to solve part of the problems, others all of them (see, e.g., [23]). These approaches are the quantum gravity approach, mass inflation, wormhole, regular black hole, holographic reasoning (see, e.g., [24]), and so on. Here, we advocate the quasiblack hole approach to better understand a black hole, both the outside and the inside stories. We do not claim to solve the problems, we look at it through a different angle and see where it leads us to.

1.4. Quasiblack holes. Following [17], for matter made of perfect fluid there is the Buchdahl limit. However, putting charge into the matter to bypass the limit opens up a new world. The charge can be electrical, or angular momentum, or many other charges. The simplest case is to have matter with electric charge alone, nothing else.

In Newtonian gravitation, i.e., for a Newton–Coulomb system, the solution is easy. Suppose one has two massive charged particles. Then, the gravitational force exerted on each particle is $F_{\rm g} = \frac{Gm^2}{r^2}$, where for a moment we have restored G, and the



Fig. 1. A star of clouds as an example of a Bonnor star: Majumdar–Papapetrou (extremal charged dust) matter inside, extremal Reissner–Nordström outside, and a boundary surface joining the inside and outside at the radius R

electric force is $F_{\rm e} = \frac{e^2}{r^2}$. Thus, when $\sqrt{G}m = e$ it implies $F_{\rm g} = F_{\rm e}$. The system is in equilibrium. Of course, if we put another such particle, any number of particles, a continuous distribution of matter, any symmetry, any configuration, the result still holds. For a continuous distribution the relation $\sqrt{G}\rho_{\rm m} = \rho_{\rm e}$ must hold, where $\rho_{\rm m}$ and $\rho_{\rm e}$ are the mass-energy density and the electric charge density, respectively.

In General Relativity, i.e., for an Einstein-Maxwell system, the history is long. In 1917 [25], Weyl started with a static solution in the form:

$$ds^{2} = -W^{2}(x^{i}) dt^{2} + g_{ij}(x^{k}) dx^{i} dx^{j}.$$
(4)

Then he sought W such that $W^2 = W^2(\phi)$, in vacuum with axial symmetry, where ϕ is the electric potential. He found $W^2 = (\sqrt{G}\phi + b)^2 + c$, with b and c constants. In 1947, Majumdar [26] showed that Weyl's quadratic function works for any symmetry, not only axial symmetry. It was also shown that the (vacuum) extremal Reissner–Nordström solution obeyed this quadratic relation, and that many such solutions could be put together since, remarkably, equilibrium would be maintained, as in the Newton–Coulomb case. Papapetrou [27] also worked along the same lines. Hartle–Hawking in 1973 [28] worked out the maximal extension and other properties of a number of extremal black holes dispersed in spacetime. Furthermore, for a perfect square, $W^2(\phi) = (\sqrt{G}\phi + b)^2$, if now there is matter, Majumdar and Papapetrou found that $\sqrt{G}\rho_m = \rho_e$ [26, 27], and the matter is in an equilibrium state, bringing into General Relativity the Newtonian result. This type of matter we call extremal charged dust matter. The solutions, vacuum or matter solutions, are generically called Majumdar–Papapetrou solutions.

Now, if one wants to make a star one has to put some boundary on the matter. The interior solution is then Majumdar – Papapetrou and the exterior is extremal Reissner – Nordström. This analysis was started by Bonnor who since 1953 has called attention to them, see, e.g., [29]. Examples of Bonnor stars are:

(i) A star of clouds, in which each cloud has 1 proton and 10^{18} neutrons, so to maintain the relation $\rho_{\rm m} = \rho_{\rm e}$ (G = 1). For a spherically symmetric star with radius R, the star as a whole has m = q, and the exterior is extremal Reissner-Nordström, see Fig. 1.

(ii) A star made of supersymmetric stable particles with $m_s = e_s$. Again, the star has total mass m and total charge q related by m = q.

Now comes the important point. For any star with radius R the star is in equilibrium. It is also in equilibrium for $R = r_{\rm h}$, where $r_{\rm h} = m$ is the gravitational, or horizon,



Fig. 2. Comparison of the generic form of the metric potentials B and A for black holes and quasiblack holes

radius of the extremal Reissner-Nordström metric. What happens when R shrinks to r_h ? Something new: a quasiblack hole forms.

2. Black hole and quasiblack hole solutions

2.1. Generic features of the solutions. The difference between an extremal spherically symmetric black hole and an (extremal) spherically symmetric quasiblack hole spacetime is best displayed if we write the metric as

$$ds^{2} = -B(r) dt^{2} + A(r) dr^{2} + r^{2} \left(d\theta^{2} + \sin^{2} \theta \, d\phi^{2} \right).$$
(5)

When one approaches the gravitational radius of the object one finds that the solutions have the features shown in Fig. 2.

For the extremal Reissner-Nordström black hole one has $B(r) = 1/A(r) = (1 - m/r)^2$, so that at $r = r_h = m$ there is the usual event horizon, and at r = 0 the potentials are singular and indeed yield a singular spacetime where the curvature invariants diverge. For the extremal quasiblack hole the function 1/A(r) is well behaved, touches zero at $r = r_h$, when a quasihorizon (not an event horizon) forms, and tends to 1 at r = 0, so that there are no conical singularities. The function B(r) is well-behaved up to the quasiblack hole limit. At the quasiblack hole limit, $R = r_h$, the function is zero in the whole interior region. This brings new features.

2.2. Black holes and quasiblack holes made of Majumdar-Papapetrou stuff. The Majumdar-Papapetrou vacuum black hole is the extremal Reissner-Nordström black hole, a solution with well-known properties.

For quasiblack holes, Majumdar-Papapetrou matter provides perhaps the simplest case, as shown by Lemos and Weinberg in 2004 [30]. In [30] a solution was found in which there is no need for a junction. In the solution, the Majumdar-Papapetrou matter decays sufficiently rapidly to yield at infinity, in a continuous way, the extremal Reissner-Nordström metric. In this way the existence of simple quasiblack hole solutions were shown beyond doubt. The potentials and all their derivatives are continuous. Thus, one avoids the possible problems caused by Bonnor stars where the potentials are only one-time differentiable. To find the solutions, we insist on putting the metric as in Eq. (5). Then Einstein-Maxwell equations give

$$\frac{(AB)'}{AB} = 8\pi r\rho A, \quad \left[r\left(1-\frac{1}{A}\right)\right]' = 8\pi r^2 \rho + \frac{r^2}{AB} {\varphi'}^2, \tag{6}$$



Fig. 3. Plots of the potentials and matter functions as a function of r for q = 1 and for four different stars, each with compact parameter c given by c = 0.5, 0.3, 0.1, 0.001. The emergence of the quasihorizon is quite evident in the c = 0.001 curve, see [30] for details

$$\frac{\sqrt{B}}{r^2\sqrt{AB}} \left[\frac{r^2}{\sqrt{AB}}\,\varphi'\right]' = -4\pi\rho_{\rm e}.\tag{7}$$

where primes denote differentiation with respect to r. One can then work out the various types of solutions [30].

These stars have no well-defined radius R since there is no boundary. The solutions tend smoothly into the extremal Reissner-Nordström vacuum. Instead, there is a compact parameter c which characterizes the solution. As this parameter tends to zero, $c \rightarrow 0$, the star gets denser at the center and more compact. At c = 0 a quasiblack hole appears. This is shown in Fig. 3, where plots for four different stars (i.e., with stars with different cs) are displayed. The one in which $c \rightarrow 0$ shows clearly a quasiblack hole behavior, with the emergence of a quasihorizon.

2.3. Other ways: Black holes and quasiblack holes made of various sorts of matter. There are black hole solutions in General Relativity other than the ones provided by the Kerr-Newman family. Those are regular black holes in which the vacuum inside the horizon with its singularity is replaced by a de Sitter core, which can be magnetically charged [31] or have non-isotropic pressures [32], or have some other form (see, e.g., [33]). There are also regular black holes electrically charged in a special way [34]. Black holes are generic.

What about quasiblack holes? Can they be built from other configurations and forms of matter other than Majumdar – Papapetrou? Yes, there are several different quasiblack hole solutions found up to now.

First, there are the simple quasiblack holes of Lemos and Weinberg, already mentioned [30].

Second, spherical Bonnor stars (charged stars with a spherical boundary surface) also yield quasiblack holes. This was shown preliminary by Bonnor himself (see, e.g., [29]) and by subsequent works [35, 36]. Moreover, recently Bonnor has shown that spheroidal stars made of extremal charged dust tend in the appropriate limit to quasiblack holes [37]. Generic properties of the Majumdar-Papapetrou matter in *d*-dimensions were displayed in [38].

Third, charged matter with pressure (with a generalized Schwarzschild interior ansatz to include electrical charge) also yields charged stars that when sufficiently compact tend to quasiblack holes. These are the relativistic charged spheres which can then be considered as the frozen stars [39–41]. For the properties of the solutions and the connection with the Weyl–Guilfoyle ansatz [42] see [43]. These solutions have additional interest since the pressure stabilizes the fluid against kinetic perturbations.

Fourth, the Einstein–Yang-Mills–Higgs equation yields gravitationally magnetic monopoles that when sufficiently compact form, in certain instances, quasiblack holes, as shown by Lue and Weinberg [44, 45]. In these works the name quasiblack hole was coined for the first time. A comparison between gravitationally magnetic monopole and Bonnor star behavior was done in [46].

Fifth, the Einstein-Cartan system with spin and torsion, in which the spinning matter, put in a spherically symmetric configuration, is joined into the Schwarzschild solution, also yields quasiblack holes [47].

Sixth, disk matter systems, when sufficiently compact and rotating at the extremal limit, have, as exterior metric, the extremal Kerr spacetime. These solutions were found by Bardeen and Wagoner back in 1971 [48]. In the new language they are quasiblack holes and their properties have been explored by Meinel and collaborators [49, 50].

Finally, it is a simple exercise to show that a shell of matter, for which the inside is a Minkowski spacetime and the outside is Schwarzschild, yields solutions with quasiblack hole properties if the shell is allowed to hover on the quasihorizon. A drawback here, that does not appear in the six cases mentioned above, is that in the quasihorizon limit the tangential pressures grow unbound. We will comment on this when we work out the mass formula for quasiblack holes.

There are certainly many other examples in which quasiblack holes may form.

3. Black holes and quasiblack holes: definition and properties

3.1. Black holes. Definition of a black hole can be seen in [51–53]. Some of the black hole properties were developed in, e.g., [54–59].

3.2. Quasiblack holes. Since it appears that quasiblack hole solutions are more ubiquitous than one could have thought, one should consider the core properties of those solutions as independently as possible from the matter they are made of, in much the same way as one does for black holes [60–66].

3.2.1. Definition. Write the metric as in Eq. (5), for an interior metric with an asymptotic flat exterior region. Consider if the solution satisfies the following requisites:

(a) The function 1/A(r) attains a minimum at some $r^* \neq 0$, such that $1/a(r^*) = \varepsilon$, with $\varepsilon \ll 1$.

(b) For such a small but nonzero ε the configuration is regular everywhere with a nonvanishing metric function B.

(c) In the limit $\varepsilon \to 0$ the metric coefficient $B \to 0$ for all $r \leq r^*$.

See Fig. 2. These three features define a quasiblack hole [60]. The quasiblack hole is on the verge of forming an event horizon, but instead, a quasihorizon appears with $r^* = r_{\rm h}$. The metric is well defined everywhere and the spacetime should be regular everywhere. One can try to give an invariant definition of a quasiblack hole instead. For instance, in (a) one can replace 1/A by $(\nabla r)^2$. Note that this definition shows that the quasihorizon is related to an apparent horizon [52] rather than to an event horizon.

3.2.2. Generic properties. A study of the several properties that can be deduced from the above definition was initiated by Lemos and Zaslavskii [60]. Some generic properties are: (i) The quasiblack hole is on the verge of forming an event horizon; instead, a quasihorizon appears. (ii) The curvature invariants remain regular

everywhere. (iii) A free-falling observer finds in his frame infinite tidal forces at the interface showing some form of degeneracy. The inner region is, in a sense, a mimicker of a singularity. (iv) Outer and inner regions become somehow mutually impenetrable and disjoint. For example, in the Lemos-Weinberg solution [30], the interior is Bertotti-Robinson, the quasihorizon region is extremal Bertotti-Robinson, and the exterior is extremal Reissner-Nordstro"m [60]. (v) There are infinite redshift whole 3-regions. (vi) For far away observers the spacetime is indistinguishable from that of black holes. (vii) Quasiblack holes with finite stresses must be extremal to the outside.

A comparison of quasiblack holes with other objects, such as wormholes, that can mimick black hole behavior was given in [61].

3.2.3. Pressure properties. One can also work out what conditions the matter pressure should obey at the boundary when the configuration approaches the quasiblack hole regime. For these interesting properties see [62].

3.2.4. The mass formula. To find the mass of a quasiblack hole, one develops the Tolman formula $m = \int (-T_0^0 + T_i^i) \sqrt{-g} d^3x$, where *i* stands for spacelike indices 1, 2, 3. Since one uses the energy-momentum tensor T_{ab} of the matter, this formula is not applicable for vacuum black holes; for black holes one has to use other methods [55, 56]. Nevertheless, in the general stationary case, we obtain in the horizon limit [63, 64]

$$m = \frac{\kappa A}{4\pi} + 2\omega_{\rm h} J + \varphi_{\rm h} q, \qquad (8)$$

where κ is the surface gravity, A is the horizon area, $\omega_{\rm h}$ is the horizon angular velocity, J is the quasiblack hole angular momentum, $\varphi_{\rm h}$ is the electric potential, and q is the quasiblack hole electrical charge. This is precisely Smarr's formula [56], but now for quasiblack holes. The contribution of the term $\frac{\kappa A}{4\pi}$ comes from the tangential pressures that grow unbound at the quasiblack hole limit but are at the same time redshifted away to give precisely $\frac{\kappa A}{4\pi}$. For the extremal case, the term $\frac{\kappa A}{4\pi}$ goes to zero, since κ is zero. See also Meinel [49, 50] for the pure stationary solution of the Bardeen–Wagoner type disks [48].

3.2.5. The entropy. To find the entropy one uses the first law of thermodynamics together with the Brown-York formalism [59]. The approach developed here is model-independent; it solely explores the fact that the boundary of the matter almost coincides with quasihorizon [65, 66].

For nonextremal quasiblack holes, when one carefully takes the horizon limit, one finds that the entropy S is [65]

$$S = \frac{1}{4}A,\tag{9}$$

where A is the quasihorizon area and is consistent with the black hole entropy [57, 58]. The contribution to this value comes again from the tangential stresses that grow unbound in the nonextremal case. Since these divergent stresses are at the boundary, the result suggests that the degrees of freedom are on the horizon. It is precisely when a quasihorizon is achieved and the system has to settle to the Hawking temperature, that the entropy has the value A/4. The result, together with the approach, suggests further that the degrees of freedom are ultimately gravitational modes. Since the tangential pressures grow unbound here, all modes, presumably quantum modes, are excited. In pure vacuum, as for a simple black hole, they should be gravitational modes.

For extremal quasiblack holes the stresses are finite at the quasihorizon. So one should deduce that not all possible modes are excited. This means that the entropy of an extremal quasiblack hole, and by continuity of an extremal black hole, should be $S \leq A/4$. Indeed, in [66] we find for extremal quasiblack holes:

$$0 < S \le \frac{1}{4}A.\tag{10}$$

The problem of entropy for extremal black holes is a particularly interesting one. Arguments based on periodicity of the Euclidean section of the black hole lead one to assign zero entropy in the extremal case. However, extremal black hole solutions in string theory typically have the conventional value given by the Bekenstein-Hawking area formula S = A/4. We find an interesting compromise.

Conclusions

Black holes are generic and stable. Quasiblack holes perhaps are not. Any perturbation would lead them into a black hole, although the inclusion of pressure may stabilize the system.

However, stable or not, the quasiblack hole approach can elucidate many features of black holes such as the mass formula and the entropy. The quasiblack hole approach to the understanding of black hole physics seems somehow similar to the membrane paradigm [67]. Indeed, by taking a time-like matter surface into a null horizon, in a limiting process, we are recovering the membrane paradigm. One big difference is that our membrane is not fictitious like the membrane of the membrane paradigm, it is a real matter membrane.

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Резюме

Х.П.С. Лемос. Черные дыры и квазичерные дыры: история и комментарии.

Приведена краткая справка о двух шварцшильдовских решениях и о том, как охарактеризовал их А.З. Петров. Обсуждается вопрос о том, как вакуумное решение Шварцшильда описывает черную дыру. Проведено сравнение свойств, различий и сходств черных и квазичерных дыр. Черные дыры хорошо известны. «Квазичерная дыра» – это новое понятие. Квазичерная дыра, как экстремальная, так и неэкстремальная, может быть грубо определена как предельная конфигурация некоего тела, при которой граница тела приближается к его собственному гравитационному радиусу (квазигоризонту). Эти объекты почти являются черными дырами, однако в действительности отличаются от них во многих отношениях. В статье представлены некоторые из свойств квазичерных дыр: существование целых областей с бесконечным красным смещением; регулярность инвариантов кривизны повсеместно в пределах квазичерной дыры; наличие внутренней области, в которой свободно падающий наблюдатель фиксирует бесконечно большие приливные силы как признак некоего вырождения; взаимная непроницаемость и несвязанность внутренних и внешних областей (однако в отличие от обычных черных дыр это разграничение имеет скорее динамический, нежели причинный характер); неразличимость пространства-времени квазичерной дыры для удаленного внешнего наблюдателя от пространства-времени экстремальной черной дыры. Рассматриваются также и другие важные свойства квазичерных дыр, такие как формула для массы и энтропия, в сравнении с их аналогами для черных дыр.

Ключевые слова: решение Шварцшильда, черные дыры, квазичерные дыры.

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