ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 3, pp. 60–64. © Allerton Press, Inc., 2013.
Original Russian Text © I.Ya. Zabotin and R.S. Yarullin, 2013, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2013, No. 3, pp. 74–79.

One Approach to Constructing Cutting Algorithms with Dropping of Cutting Planes

I. Ya. Zabotin* **and R. S. Yarullin****
(Submitted by Ya.I. Zabotin)

Kazan (Volga Region) Federal University, ul. Kremlyovskaya 18, Kazan, 420008 Russia

Received July 19, 2012

Abstract—We propose a general cutting method for conditional minimization of continuous functions. We calculate iteration points by partially embeddimg the admissible set in approximating polyhedral sets. We describe the features of the proposed method and prove its convergence. The constructed general method does not imply the inclusion of each of approximating sets in the previous one. This feature allows us to construct cutting algorithms which periodically drop any additional restrictions which occur in the solution process.

DOI: 10.3103/S1066369X13030092

Keywords and phrases: *conditional minimization, approximating set, cutting plane, algorithm, sequence of approximations, convergence.*

Cutting methods represent a peculiar class of conditional minimization methods (e.g., [1–4]). They are united by the following common idea. On each step of a method, one constructs an iteration point using a convex set which approximates the initial feasible set but has a simpler structure. Each next approximating set is constructed on the base of the previous one by cutting (usually, by hyperplanes) of some subset which contains the current iteration point.

One of problems which arise in the numerical realization of such methods consists in the fact that the number of cutting hyperplanes indefinitely increases from step to step. Therefore, so does the number of inequalities that define approximating sets. In this connection, the complexity of finding iteration points grows with the growth of the step number.

One can save such methods from the mentioned imperfection by dropping accumulated constraints in accordance with some principle. Such a technique is applied, for example, in the algorithm proposed in [1] (P. 30), but its convergence is proved only for a strongly convex objective function.

In the present paper we propose a dropping principle applicable for problems with nearly all objective functions. It is used in the general cutting method proposed here; this method does not require that each approximating set should be included in the previous one. This principle is based on the fact that a certain proximity of an iteration point to the initial feasible set allows one to choose the next approximating set almost arbitrarily. Therefore it is possible to construct algorithms, where one periodically drops an arbitrary number of any additional constraints obtained in the solution process.

Let $f_j(x)$, $j = 1, \dots, m$, be convex functions defined in an n-dimensional Euclidean space R_n, $J = \{1, \dots, m\}$,

$$D' = \{x \in R_n : f_j(x) \le 0,\ j \in J\},$$

and let $D'' \subset R_n$ be a convex closed set,

$$D = D' \bigcap D''.$$

We assume that for each $j \in J$ the set

$$D_j = \{x \in R_n : f_j(x) \le 0\}$$

*E-mail: IYaZabotin@mail.ru.
**E-mail: YarullinRS@gmail.com.