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The Global Search in the Tikhonov Scheme

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Abstract—The goal of this paper is to justify a general scheme for constructing two-stage iterative solution processes for irregular nonlinear operator equations based on the sequential approximate minimization of locally strongly convex Tikhonov functionals.

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1. INTRODUCTION

Consider a nonlinear operator equation

$$F(x) = f, \tag{1}$$

where the operator $F : H_1 \to H_2$ is Frechet differentiable, H_1 and H_2 are Hilbert spaces, and $f \in H_2$. Assume that Eq. (1) has a solution x^*, and

$$\|F'(x)\| \le N_1, \quad \|F'(x) - F'(y)\| \le N_2\|x - y\| \quad \forall x, y \in \Omega_R(x^*). \tag{2}$$

Hereinafter $\Omega_R(x) = \{y \in H_1 : \|y - x\| \le R\}$ and the symbol $\|\cdot\|$ denotes the norm of the corresponding space. In addition, we assume that the operator F is weakly closed, i.e., for any sequence $\{x_n\} \subset H_1$ the weak convergence $x_n \to x$ in H_1 and the weak convergence $F(x_n) \to y$ in H_2 in aggregate imply the equality $y = F(x)$.

Since except for requirements of the smoothness and the weak closedness no other structural restrictions like the regularity or monotonicity (for example, [1, 2]) are imposed on the operator F, problem (1) in a general case is ill-posed. In practice this means that if the operator F or the element f are defined inexactly, one needs regularization methods for approximately solving (1). Let us restrict ourselves to the case when the operator F is defined exactly, but instead of the element f in (1) we know only its approximation f_δ such that $\|f_\delta - f\| \le \delta$. A. N. Tikhonov [1, 3] has proposed a universal approach to constructing numerical methods for the stable solution of Eq. (1) under the presence of errors. Let us describe one of the most known techniques that realize this approach. One constructs the Tikhonov functional

$$\Phi_\alpha(x) = \frac{1}{2}\|F(x) - f_\delta\|^2 + \frac{1}{2}\alpha\|x - \xi\|^2, \quad x \in H_1, \tag{3}$$

where $\alpha > 0$ is a regularization parameter and $\xi \in H_1$ is an a proiri approximation of the desired element x^*. As an approximation of the solution x^* one uses an exact or approximate solution to the problem

$$\min_{x \in H_1} \Phi_\alpha(x). \tag{4}$$

The existence of a solution x_α to problem (4) is guaranteed by the weak closedness of the operator F and the correlation $\lim\limits_{\|x\| \to \infty} \Phi_\alpha(x) = \infty \ \forall \alpha > 0$. Under the proper additional conditions that include

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