Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41, No. 10, pp. 79–81, 1997 *UDC 517.984*

ON A PEARCY-SHIELDS PROBLEM

N.B. Uskova

In the paper by C. Pearcy and A. Shields [1] it was shown that if A, B are complex $n\times n-$matrices, $A = A^*$, and the condition

$$\|AB - BA\| \le \frac{2\varepsilon^2}{n-1} \tag{1}$$

holds, then there exist matrices A', B' of the same dimensions such that $A'B' = B'A'$ and $\|A-A'\| \le \varepsilon$, $\|B - B'\| \le \varepsilon$. Numerous references may be cited for a similar problem for bounded operators (see, e. g., [1], [4]). In the papers by G.V. Rosenblyum [2], [3], S.Yu. Sadov [4] a question was posed: Given a pseudodifferential operator, how close it is to a normal one? There was also posed the question about the construction of the commuting approximants.

In the present article we consider pairs of linear operators $A, B : D(A) \subset H \to H$ acting in a complex Hilbert space H and having a domain $D(A)$, A being a selfadjoint operator with a countable spectrum $\sigma(A)$ which consists of eigenvalues $\lambda_1, \lambda_2, \dots$ and allows a representation in the form

$$\sigma(A) = \bigcup_{j=1}^{N} \sigma_j, \tag{2}$$

where σ_j, $j = 1, 2, \dots, N \le \infty$ are disjoint bounded compacts.

Everywhere in what follows the next assumption takes place.

Assumption 1. *For any $\varepsilon > 0$, there exists a partition $\mathfrak{A}$ of the spectrum $\sigma(A)$ of A of the form (2), such that the following two conditions hold*: 1) $\sup\limits_{1\le j\le N} \operatorname{diam}\sigma_j \le 2\varepsilon$, 2) $d(\varepsilon, \mathfrak{A}) = \inf\limits_{i\ne j} \operatorname{dist}(\sigma_i, \sigma_j) > 0$.

A partition $\mathfrak{A}$ from Assumption 1 will be called *admissible.* Note that the conditions of Assumption 1 are satisfied by the compact operators and operators with a discrete spectrum, therefore there exists a positive integer n_0 such that each segment $[n, n+1]$ contains no more than n_0 eigenvalues.

Consider a function $d(\varepsilon) = d(\varepsilon, A)$, $\varepsilon > 0$, by setting $d(\varepsilon) = \sup\limits_{\mathfrak{A}} d(\varepsilon, \mathfrak{A})$ taken over all admissible partitions of $\mathfrak{A}$.

In addition to Assumption 1, we also admit the validity of the following

Assumption 2. *The operator $B : D(A) \subset H \to H$ is subordinate to A (i. e., a constant $c > 0$ exists such that $\|Bx\| \le c(\|x\| + \|Ax\|)$ $\forall x \in D(A)$) and the commutator $C = [A, B] = AB - BA : D(A^2) \subset H \to H$ has a bounded extension to an operator from the Banach algebra* End H *of the bounded linear operators acting in H.*
