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A Nonlocal Problem with Fractional Derivatives for the Mixed Type Equation

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Abstract—For an equation of the mixed elliptic-hyperbolic type we investigate the boundary-value problem, when on the elliptic part of domain boundary a co-normal derivative of solution is given, and in the hyperbolic part the generalized fractional derivatives of solution value on characteristics are pointwise connected with the solution value and its derivative on the line of parabolic degeneration. The unique resolvability of problem is proven.

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Let us consider a mixed type equation

$$\operatorname{sgn} y|y|^m u_{xx} + u_{yy} = 0, \quad m = \text{const} > 0, \tag{1}$$

in a finite domain Ω, which is bounded by a smooth Jordan curve σ with ends at points $A(0,0)$, $B(1,0)$, which is contained in a half-plane $y > 0$, and characteristics

$$AC : x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0, \quad BC : x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$$

of Eq. (1).

Let Ω_1 and Ω_2 be the elliptic and hyperbolic parts of the mixed domain Ω.

We mean that the regular solution to Eq. (1) is the function $u(x,y)$ from the class $\mathbb{C}(\overline{\Omega}) \cap \mathbb{C}^1(\Omega) \cap \mathbb{C}^2(\Omega_1 \cup \Omega_2)$, which satisfy Eq. (1) in $\Omega_1 \cup \Omega_2$ and such that derivatives $u_x(x,0)$, $u_y(x,0)$ can become infinity of order $\frac{2}{m+2}$ at the ends A and B of interval $J : 0 < x < 1$ of the right line $y = 0$.

Problem. *Find a solution $u(x,y)$ to Eq. (1) regular in the domain Ω, which satisfies conditions*

$$A_s[u]|_\sigma \equiv y^m \frac{dy}{ds}\frac{\partial u}{\partial x} - \frac{dx}{ds}\frac{\partial u}{\partial y}\Big|_\sigma = \varphi(s), \quad 0 < s < l, \tag{2}$$

$$a(x)\big(I_{0+}^{\alpha_1,\beta_1,\eta_1}\delta(t)u[\theta_0(t)]\big)(x) + b(x)(I_{1-}^{\alpha_2,\beta_2,\eta_2}\omega(t)u[\theta_1(t)])(x) + c(x)u(x,0) + d(x)u_y(x,0) = g(x) \ \forall x \in J, \tag{3}$$

where α_i, β_i, and η_i $(i = 1,2)$ are real numbers, s is the length of curve σ, which is measured from the point B; $\theta_0(x)$, $\theta_1(x)$ are points of intersection of characteristics of Eq. (1), which go from point $(x,0) \in J$, with characteristics AC, BC, respectively; $\varphi(s)$, $a(x)$, $b(x)$, $c(x)$, $d(x)$, $g(x)$, $\delta(x)$, $\omega(x)$ are a given function such that

$$\varphi(s) \in \mathbb{C}^1(\sigma), \quad a(x), b(x), c(x), d(x), g(x) \in \mathbb{C}^1(\overline{J}), \tag{4}$$

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