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Lepage Forms, Closed 2-Forms and Second-Order Ordinary Differential Equations

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Abstract—Lepage 2-forms appear in the variational sequence as representatives of the classes of 2-forms. In the theory of ordinary differential equations on jet bundles they are used to construct exterior differential systems associated with the equations and to study solutions, and help to solve the inverse problem of the calculus of variations: since variational equations are characterized by Lepage 2-forms that are closed. In this paper, a general setting for Lepage forms in the variational sequence is presented, and Lepage 2-forms in the theory of second-order differential equations in general and of variational equations in particular, are investigated in detail.

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1. INTRODUCTION

Ordinary differential equations in jet bundles can be studied with help of Krupka's variational sequence [13, 14], and of differential forms that appear as representatives of classes in the sequence. In this paper we consider differential equations represented by *dynamical forms* on jet-prolongations of fibered manifolds [18, 23, 25, 26]). In Section 4 we study different representations of classes in the variational sequence; in particular, we present a generalisation to the concepts of *source forms* and *Lepage forms*, and we study Lepage equivalents of dynamical forms. In Section 5 Lepage forms are used to construct exterior differential systems associated with second-order differential equations, and to provide different classifications of the equations: 1) with respect to the properties of Lepage equivalents of dynamical forms that "measure" variationality (existence of Lagrangians), and 2) with respect to the properties of the solutions. In the classification, namely *regular* equations on one hand, and *semivariational* and *variational* equations on the other hand, are important distinguished classes of equations. The last two Sections are devoted to regular equations. Following Krupková [19, 23] we present their geometric descriptions by means of semisprays and semispray connections, and study in detail Lepage forms corresponding to regular dynamical forms and semisprays. We focus on the so-called Kähler lift Ω of a regular symmetric $(0, 2)$-tensor field g, discovered by Crampin, Prince and Thompson [4] and clarify the place of this 2-form in the variational sequence and in the general theory of Lepage forms. We also recall the role of this form in the solution of the inverse variational problem for semisprays.

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