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Fractal Approximation of Vector Functions

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Abstract—We present a new approach to the approximation of continuous vector-valued functions by fractal interpolative vector-valued ones and find optimal values of their parameters. We illustrate the obtained results with examples.

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1. INTRODUCTION

It is well-known that curves such as shorelines, currency fluctuation graphs, encephalograms, and many others are, in fact, fractals, because their Hausdorff–Besicovitch dimension exceeds 1. One can approximate these curves by fractal interpolation ones [1] and their various generalizations ([2], P. 182) rather than by graphs of classical smooth functions (like polynomials or splines).

This work is a multi-dimensional generalization of [3]. We consider fractal interpolation vector functions depending on several parameter matrices and pose an optimization problem for approximating a given continuous vector function by a fractal interpolation one. Using matrix differentiation methods, we find optimal values of parameter matrices.

2. FRACTAL INTERPOLATION VECTOR FUNCTIONS

Let $[a,b] \subset \mathbb{R}$ be a nonempty interval, $1 < N \in \mathbb{N}$, and let $\{(t_n, \mathbf{x}_n) \in [a,b] \times \mathbb{R}^M \mid a = t_0 < t_1 < \cdots < t_{N-1} < t_N = b\}$ be interpolation nodes. Now for any $n = \overline{1,N}$ we consider the affine transform

$$A_n : \mathbb{R}^{M+1} \to \mathbb{R}^{M+1}, \quad A_n \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} := \begin{pmatrix} a_n & \mathbf{0} \\ \mathbf{c}_n & \mathbf{D}_n \end{pmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} + \begin{pmatrix} e_n \\ \mathbf{f}_n \end{pmatrix}. \tag{1}$$

Here and in what follows we use bold lowercase letters for denoting columns (rows) of length M and we do bold uppercase letters for denoting matrices of order $M \times M$.

Assume that the following conditions are fulfilled for any n:

$$A_n(t_0, \mathbf{x}_0) = (t_{n-1}, \mathbf{x}_{n-1}), \quad A_n(t_N, \mathbf{x}_N) = (t_n, \mathbf{x}_n).$$

Hence

$$\begin{aligned} a_n &= \frac{t_n - t_{n-1}}{b-a}, & e_n &= \frac{bt_{n-1} - at_n}{b-a}, \\ \mathbf{c}_n &= \frac{\mathbf{x}_n - \mathbf{x}_{n-1} - \mathbf{D}_n(\mathbf{x}_N - \mathbf{x}_0)}{b-a}, & \mathbf{f}_n &= \frac{b\mathbf{x}_{n-1} - a\mathbf{x}_n - \mathbf{D}_n(b\mathbf{x}_0 - a\mathbf{x}_N)}{b-a}, \end{aligned} \tag{2}$$

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