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CONDITION OF SOURCE-REPRESENTABILITY AND ESTIMATES FOR CONVERGENCE RATE OF METHODS FOR REGULARIZATION OF LINEAR EQUATIONS IN BANACH SPACE. II

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1. The necessity of the condition of source-representability

In this article which continues [1] we consider the question about the necessity of sourcewise representation of the initial discrepancy for the power estimate of the convergence rate of the method of approximation of solutions of the linear ill-posed equations

$$Ax = f, \quad x \in X. \tag{1.1}$$

As in the preceding text, here X is a complex Banach space with the norm $\|\cdot\|$, $A \in \mathcal{L}(X)$, $f \in X$; $\mathcal{L}(X)$ is the space of linear continuous operators acting from X to X. The norm in $\mathcal{L}(X)$ is introduced in the standard way and denoted by $\|\cdot\|_{\mathcal{L}(X)}$. The existence and continuity of the inverse operator A^{-1} is not supposed which turns equation (1.1) into ill-posed. In this article, for the sake of simplicity we assume that the initial data (A, f) in (1.1) are known without errors. We are speaking about the class of methods

$$x_\alpha = (E - \theta(A,\alpha)A)\xi + \theta(A,\alpha)f, \quad \alpha \in (0, \alpha_0]; \tag{1.2}$$

within the frameworks of them concrete procedures are determined by the choice of generating complex-valued functions $\theta(\lambda,\alpha)$, $\lambda \in \mathbb{C}$, $\alpha \in (0, \alpha_0]$. Here E is the unit operator, α the regularization parameter, $\xi \in X$ a fixed element serving as an initial approximation to the desired solution x^*. It is assumed that the set of solutions of (1.1) is nonempty. The function $\theta(A,\alpha)$ of the operator A in (1.2) is defined by the Riesz–Dunford formula

$$\varphi(A) = \frac{1}{2\pi i}\int_\Gamma \varphi(\lambda)R(\lambda, A)d\lambda, \tag{1.3}$$

where $R(\lambda, A) = (\lambda E - A)^{-1}$ stands for the resolvent of the operator A, Γ is a positively oriented contour on the complex plane $\mathbb{C}$, which embraces the spectrum $\sigma(A)$ of the operator A and lies entirely in the domain of analyticity of the function $\varphi(\lambda)$.

Let us recall the basic conditions upon the operator A and the generating functions $\theta(\lambda,\alpha)$, under which in [1] the approximation properties of scheme (1.2) with respect to the desired solution x^* were considered.

Condition A. For a certain $\varphi_0 \in (0, \pi)$, the following inclusion is fulfilled

$$\sigma(A) \subset K(\varphi_0), \quad K(\varphi_0) = \{\lambda \in \mathbb{C} : |\arg\lambda| \le \varphi_0\} \tag{1.4}$$

as well as the estimate

$$\|R(\lambda, A)\|_{\mathcal{L}(X)} \le \frac{c_0}{|\lambda|} \quad \forall \lambda \in \mathbb{C} \setminus K(\varphi_0). \tag{1.5}$$