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On Exact Sufficient Oscillation Conditions for Solutions of Linear Differential and Difference Equations of the First Order With Aftereffect

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Abstract—We obtain new unimprovable effective oscillation conditions for all solutions of linear first-order differential and difference equations with several delays. We show that known results of the kind are consequences of the new results. We reveal the reasons for the impossibility to obtain oscillation conditions for equations with several delays, as sharp as the conditions for the equation with one delay, in the case when only known approaches are used.

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INTRODUCTION

Consider a differential equation with an alternating delay:

$$\dot{x}(t) + p(t)x(\tau(t)) = 0, \quad t \in \mathbb{R}_+, \tag{1}$$

where functions $p, \tau : \mathbb{R}_+ \to \mathbb{R}$ are continuous, $p(t) \geq 0$, $\tau(t) \leq t$ for all $t \in \mathbb{R}_+$, and $\tau(t) \to +\infty$ for $t \to +\infty$.

In 1982, R. G. Koplatadze and T. A. Chanturiya published the following theorem which generalizes oscillation conditions for the solutions of Eq. (1) obtained by A. D. Myshkis [1] and G. Ladas [2], and unites the merits of these results.

Theorem 1 ([3]). *All solutions to Eq.* (1) *oscillate, provided that* $\varliminf_{t\to\infty} \int\limits_{\tau(t)}^{t} p(s)\,ds > 1/e.$

Theorem 1 connects the oscillation of solutions to Eq. (1) with the estimate of lower limit of values of a function defined by means of parameters of this equation. There are conditions of oscillation in the form of estimates of upper limit of values of similar function.

Theorem 2 ([4–6]). *Let* $\sigma(t) = \sup\limits_{s\in[0,t]} \tau(s)$. *Then all solutions to Eq.* (1) *oscillate, provided that*

$$\varlimsup_{t\to+\infty} \int\limits_{\sigma(t)}^{t} p(s)ds > 1.$$

In Theorems 1 and 2, the strict inequalities cannot be changed to non-strict ones.

This paper is devoted to generalization of Theorems 1 and 2 in the case of differential equation with several delays and to similar problem for difference equations with aftereffect.

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