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ON SPECTRAL PROPERTIES OF RELATIVELY FINITE-DIMENSIONAL PERTURBATIONS OF SELFADJOINT OPERATORS

E.L. Ul'yanova

Let H be an infinite-dimensional complex Hilbert space and $A : D(A) \subset H \to H$ be a selfadjoint semibounded discrete operator (i. e., $R(\lambda, A) = (A - \lambda I)^{-1}$, where $\lambda \in \rho(A)$ is a compact operator) whose eigenvalues $0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n < \cdots$ corresponding to the eigenvectors $e_1, e_2, \dots, e_n, \dots$ are simple.

By using the similar operators method (see [1]–[3]) the spectral properties of the perturbed operators

$$A - B, \quad Bx = \sum_{m=1}^{N} (Ax, a_m) b_m, \quad x \in D(A), \tag{1}$$

are studied, where a_m, b_m, $m = 1, \dots, N < \infty$ are sets of vectors from H. Such a perturbation B is called *relatively finite-dimensional.* Estimates for both eigenvalues and eigenvectors of operators of the form (1) are obtained.

The similar operators method is based on the notion of an admissible triple $(\mathfrak{A}, J, \Gamma)$ (see [2]). Here $\mathfrak{A}$ is the space of admissible perturbations, which contains the operator B, and $J : \mathfrak{A} \to \mathfrak{A}$, $\Gamma : \mathfrak{A} \to \operatorname{End} H$ are two linear transformers (i. e., linear operators acting in a space of linear operators).

Let n be a positive integer, $P_n x = (x, e_n) e_n$ be the Riesz projector corresponding to λ_n. In the capacity of the space of perturbations we take an operator lineal from the Banach space $L_A(H)$ of linear operators, subordinate to A, and representable as

$$X = X_0 A, \quad X \in \sigma_2(H), \tag{2}$$

where $\sigma_2(H)$ is the ideal of Hilbert-Schmidt operators such that

$$\|P_i X P_j\| \le \operatorname{const} \alpha_i \beta_j \ \forall i, j, \tag{3}$$

nonzero sequences $\{\alpha_i\}$, $\{\beta_j\}$ of nonnegative numbers belong to the space l_2 and are so chosen that the operator B satisfies condition (3). In particular, (3) holds, whenever

$$\alpha_i = \Bigl(\sum_{m=1}^{N} |(e_i, a_m)|^2 \Bigr)^{1/2}, \qquad \beta_j = \Bigl(\sum_{m=1}^{N} |(e_i, b_m)|^2 \Bigr)^{1/2}. \tag{4}$$

We convert $\mathfrak{A}$ into a normed space by setting

$$\|X\|_* = \inf\{c > 0 : \|P_i X_0 P_j\| \le c \alpha_i \beta_j \ \forall i, j\}. \tag{5}$$