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TWO-LEVEL METHODS FOR SPATIAL PROBLEMS, BASED ON AGGREGATION

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1. Introduction

In many cases multigrid and multilevel methods possess an optimal rate of convergence and the computational work required to obtain any fixed accuracy is proportional to the number of unknowns. The principle idea of the multigrid, unchanged from the pioneering works of Fedorenko (see [1], [2]) and Bakhvalov (see [3]) consists of capturing of oscillatory response of the system by means of smoothing iterations on the fine grid. Whereas the remaining lower frequency response is resolved on an auxiliary coarse grid. Although multigrid-multilevel methods and multigrid-multilevel preconditioners in CG and other iterative techniques, as well as various aspects influencing their efficiency, have been rather thoroughly studied, their use in computational structural analysis is less wide, than it could be expected. The reason is well-known: In order to effectively apply multigrid methods, commercial software packages must be able to produce a full sequence of auxiliary discretizations that are gradual coarsenings of the source discretization. Moreover, smooth solution components relative to a given discretization must be well approximated by subsequent coarser grids. This may be a problem for many modern structures, because in the contemporary numerical practice each substucture of them is often represented by a single or a few elements.

These difficulties motivated the development of multi- and two-level techniques, which are more adapted to engineering coarsening procedures, and, in particular, methods of *aggregation* (see [4]–[16]). While the classic, called also *geometric*, multigrid approach creates the sequence of discretizations on the basis of auxiliary triangulations, the aggregation approach attempts to accomplish the same goal on the basis of the information about the matrix of the source system of algebraic equations. In many cases these methods have been found to be far more efficient, than the state-of-the-art direct solvers (see. e. g., [15], [16]). The possibility of arranging aggregates in the framework of the multilevel approach seems to be very useful in itself, since small subregions with stiffness characteristics considerably different from the rest of the structure are often met in applications and may be used as natural units for specifying aggregates.

In the simplest form, termed here as *simple aggregation*, all nodes of the fine grids are subdivided into disjoint, connected subsets — aggregates — and, thus, aggregates are separated by a single layer of elements. Only constant field or, in general, fields with zero energy are admissible on each aggregate, as, for instance, in [9], [10]. Although it is often used in practice, the simple aggregation suffers from the two obvious interrelated shortcomings: a) it does not provide a proper reduction of the maximal eigenvalue, and b) it fails to adequately approximate the lowest frequencies on coarser levels. The consequence of this is that only the weak approximation property and weak convergence estimates may be established, see Section 4.