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PROJECTIVE LAYER-LATTICES OF SMALL GEOMETRIC DIMENSION

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A finite modular lattice $\mathcal{L}$ is said to be primary (see [1]) if

1) every element X of $\mathcal{L}$ is both the sum of cycles contained in X and the product of dual cycles containing X;

2) every interval in $\mathcal{L}$ that is not a chain contains at least three atoms.

Every element of a primary lattice $\mathcal{L}$ can be represented in the form of a sum of independent cycles. If $L = A_1 + A_2 + \cdots + A_n$ is the representation of the unit, then n is called the dimension of $\mathcal{L}$. Next, if d is the least number such that $l(A_1) = l(A_2) = \cdots = l(A_d)$, then d is called the geometric dimension of the primary lattice $\mathcal{L}$. If X is an element of $\mathcal{L}$ and X^* is the sum of all elements covering X, then the interval $[X, X^*]$ is a complemented lattice. Such an interval will be called a layer of $\mathcal{L}$. A primary lattice such that every layer is a projective geometry over the same field $GF(p^k)$, provided that the layer satisfies Desargues' law, will be called a layer-projective lattice.

In [2], the definition of the layer-projective lattice contains the requirement that this lattice must admit an involutive automorphism. In fact, we do not need such a strong requirement, therefore in this article the definition of layer-projective lattice is weakened.

Since a projective geometry of dimension not less than three satisfies Desargues' law, any primary lattice of dimension no less than four is layer-projective.

Let $\mathcal{L}$ be a layer-projective lattice, $L = A_1 + A_2 + \cdots + A_n$ be a representation of unit as the sum of independent cycles, and $l(A_i) = m_i$, $m_1 \geq m_2 \geq \cdots \geq m_n$. Suppose that every layer of lattice $\mathcal{L}$ is a Desarguesian projective geometry over $GF(p^k)$. Then $(m_1, m_2, \ldots, m_n, p^k)$ is called the type of $\mathcal{L}$.

In this article we proceed with the investigation of the isomorphism problem for layer-projective lattices of same type, which was started in [2]. Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be layer-projective lattices of the same type, d the geometric dimension of these lattices.

The following results are known.

1) For $d \geq 3$, if $\mathcal{L}_1$, $\mathcal{L}_2$ are Arguesian lattices, then $\mathcal{L}_1 \cong \mathcal{L}_2$.

This follows from the main result in [1].

2) If $d \geq 4$, then $\mathcal{L}_1$, $\mathcal{L}_2$ are Arguesian lattices (see [3]); consequently, $\mathcal{L}_1 \cong \mathcal{L}_2$.

Thus, to solve the isomorphism problem for layer-projective lattices of the same type we can restrict our consideration to lattices of a small geometric dimension. For these lattices it is known that

3) if $n = 2$, then $\mathcal{L}_1 \cong \mathcal{L}_2$ (this was proved in [4] for $k = 1$, but the proof also remains valid for $k > 1$);

4) if $m_2 = m_3 = \cdots = m_n = 1$, then $\mathcal{L}_1 \cong \mathcal{L}_2$ (see [2]).

In general, the isomorphism problem for layer-projective lattices of same type does not admit a positive solution. Namely, in [3], an example was constructed which demonstrated that nonisomorphic layer-projective lattices of same type $(2, 2, 1, p^k)$ exist.