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EXTREMAL METHOD FOR SOLVING PARAMETRIC INVERSE PROBLEM FOR A SYSTEM OF LINEAR FUNCTIONAL EQUATIONS

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Introduction

Let $\mathbf{H}$ be a Hilbert space, $\{A_{\eta k}\}_{\eta\in D}$, $k = 1,2,\dots,n$, parametric families of linear operators acting in $\mathbf{H}$; $D \subset \mathbf{R}^m$ a given finite-dimensional compact. In this article we consider the problem on determination of a vector of parameters $\eta \in D$ along the system of linear functional equations

$$A_{\eta k}x = u_k, \quad k = 1,2,\dots,n, \tag{1}$$

with given $u = (u_1, u_2, \dots, u_n) \in \mathbf{H}^n$ and unknown $x \in \mathbf{H}$ (see [1]). In the capacity of a solution of the stated problem we take

$$\eta^* = \arg\min_{\eta\in D}\Big(\inf_{x\in\mathbf{H}}\sum_{k=1}^{n}\|A_{\eta k}x - u_k\|^2\Big). \tag{2}$$

Actually, the considered approach represents a development of the quasisolution method (see [2], [3]) to the case where the operators $A_{\eta k}$, $k = 1,2,\dots,n$, may be irreversible and their kernels may depend on the desired vector η. As we shall prove in what follows, imposing natural constraints upon the operators $A_{\eta k}$, $k = 1,2,\dots,n$, ensures the stability of problem (2) with respect to perturbations of right sides $u_k \in \mathbf{H}$ of system of equations (1). In the statement of problem (2) no necessity of calculation of $x \in \mathbf{H}$ is present; however, explicit presence of x turns problems (1)–(2) into infinite-dimensional and difficult for solving. In this article we investigate a method for transformation of problem (2) to a problem without the unknown x (i. e., to the mathematical programming problem) in the case where the operators $A_{\eta k}$ are functions of the selfadjoint operator T acting in $\mathbf{H}$.

Reduction to problem of mathematical programming

First we suppose the existence of an element $y \in \mathbf{H}$, which supplies the infimum in the expression (2). Since the quantity

$$\sum_{k=1}^{n}\|a_k\|^2, \quad a_1, a_2, \dots, a_n \in \mathbf{H},$$

represents the square of the norm in the Hilbert space $\mathbf{H}^n$, we have that $(A_{\eta 1}y, A_{\eta 2}y, \dots, A_{\eta n}y)$ is the orthogonal projection of the vector $u\in H^n$ onto the linear manifold $A_{\eta 1}\mathbf{H}\times A_{\eta 2}\mathbf{H}\times\cdots\times A_{\eta n}\mathbf{H} \subset H^n$, i. e.,

$$(\forall x \in \mathbf{H})\Big(\sum_{k=1}^{n}(A_{\eta k}y - u_k, A_{\eta k}x) = 0\Big),$$