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PHASE SPACE OF A NON-CLASSIC MODEL

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In [1], for the equation

$$u_t - \chi u_{xxt} = \nu u_{xx} - u u_x, \tag{0.1}$$

the Cauchy–Dirichlet problem was considered under the assumption that $\chi, \nu \in \mathbb{R}_+$. Equation (0.1) is a one-dimensional analog of the Oskolkov system [2] and can be considered as a hybrid of the Benjamin–Bona–Mahony equation and the Burgers equation [3]. The Oskolkov system

$$(1 - \chi \triangledown^2) u_t = \nu \triangledown^2 u - (u \cdot \triangledown) u - \triangledown p + f, \quad \triangledown \cdot u = 0$$

describes the dynamics of incompressible viscoelastic Kelvin–Voigt fluid. In the one-dimensional case, (0.1) models the flow of incompressible viscoelastic flow in a pipe. We will consider the case of fluid flow in a pipeline.

Let $\mathbf{G} = \mathbf{G}(\mathfrak{V}; \mathfrak{E})$ be a finite oriented connected graph, where $\mathfrak{V} = \{V_i\}$ is the set of vertices, $\mathfrak{E} = \{E_j\}$ is the set of arcs, and each arc has length $l_j > 0$ and width $d_j > 0$. On the graph $\mathbf{G}$ we consider the problem with boundary conditions

$$u_j(0,t) = u_k(l_k,t), \quad E_j, E_k \in E^{\alpha}(V_i) \cup E^{\omega}(V_i), \tag{0.2}$$

$$\sum_{E_j \in E^{\alpha}(V_i)} d_j u_{jx}(0,t) - \sum_{E_k \in E^{\omega}(V_i)} d_k u_{kx}(l_k,t) = 0 \tag{0.3}$$

and initial conditions

$$u_j(x,0) = u_{0j}(x), \quad x \in (0, l_j) \tag{0.4}$$

for the equations

$$u_{jt} - \chi u_{jxxt} = \nu u_{jxx} - u_j u_{jx}, \quad x \in (0, l_j), \quad t \in \mathbb{R}. \tag{0.5}$$

Here we denote by $E^{\alpha(\omega)}(V_i)$ the set of arcs whose start, or end, point is V_i. Condition (0.2) requires that the solutions are continuous at the vertices of the graph. The condition (0.3), an analog of the Kirkhoff condition, turns into the Neumann condition if the graph $\mathbf{G}$ consists of a unique noncyclic arc.

Study of boundary and initial problems for partial differential equations given on graphs has been recently begun [4], [5]. By now, these equations have been intensively investigated in various aspects [6], [7]. A Sobolev type equation on a graph was considered for the first time in [8]. The aim of the present paper is to find when the problem (0.2)–(0.5) has unique solution, using the approach developed in [8]. The theory of Sobolev type equations is a wide field of investigation in mathematics. Recent advances in this field are described in [9]–[12]. Unlike these papers, we use the method of phase space based on the theory of relatively σ-based operators and degenerate