ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 3, pp. 38–45. © Allerton Press, Inc., 2013.

*

Some Fixed Point Theorems for Weakly T-Chatterjea and Weakly T-Kannan-Contractive Mappings in Complete Metric Spaces

A. Razani** and **V. Parvaneh*****

Department of Mathematics, Islamic Azad University, Karaj Branch, Karaj, Iran

Received January 17, 2012

Abstract—In this work we introduce the notions of generalized weakly T-Chatterjea-contractive and generalized weakly T-Kannan-contractive maps. For these classes of maps we obtain sufficient conditions for the existence of a unique fixed point in a complete metric space.

DOI: 10.3103/S1066369X13030055

Keywords and phrases: *fixed point, complete metric space, weak C-contraction, altering distance function.*

1. INTRODUCTION AND PRELIMINARIES

The theoretical framework of fixed point theory has been an active research field over the last decade. Of course, the Banach contraction mapping principle [1] is the first important result on fixed points for contractive-type mapping. This well-known theorem which is a very essential tool in many branches of mathematical analysis, first appeared in explicit form in Banach's thesis in 1922, where it was used to establish the existence of a solution for an integral equation. So far, there have been many theorems dealing with mappings satisfying various types of contractive inequalities.

Throughout this paper, X is assumed to be a nonempty set.

The concepts of T-contraction and C-contraction have been introduced by Kannan [7] and Chatterjea [3] as follows.

Definition 1.1. a) A mapping $T : X \to X$, where (X, d) is a metric space, is said to be a C-contraction (see [3]) if there exists $\alpha \in (0, \frac{1}{2})$ such that for all $x, y \in X$ the following inequality holds:

$$d(Tx, Ty) \le \alpha\big(d(x, Ty) + d(y, Tx)\big).$$

b) A mapping $T : X \to X$, where (X, d) is a metric space, is said to be a K-contraction (see [7]) if there exists $\alpha \in (0, \frac{1}{2})$ such that for all $x, y \in X$ the following inequality holds:

$$d(Tx, Ty) \le \alpha\big(d(x, Tx) + d(y, Ty)\big).$$

In 1972, Chatterjea [3] has proved that if (X, d) is a complete metric space, then every C-contraction on X has a unique fixed point. Also, in 1968 Kannan [7] had established a fixed point theorem for a K-contraction.

*The text was submitted by the authors in English.

**E-mail: razani@ipm.ir.

***E-mail: vahid.parvaneh@kiau.ac.ir.