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APPLICATION OF IMPLICIT ALGORITHMS OF THE METHOD OF SOLUTION CONTINUATION FOR NUMERIC INTEGRATION OF DYNAMIC SYSTEMS

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In this paper, we consider the numeric solution of the Cauchy problem for systems of ordinary differential equations of the second order. We prove that one can construct simple efficient implicit computational algorithms of the stepwise integration without any laborious iteration procedures based on the processes of the Newton–Raphson iteration type. As a preliminary, one should modify the initial problem, proceeding to a new argument, namely, the length of the integral curve. This transformation is performed by the equation which connects the initial argument of the problem with the length of the integral curve. The linear acceleration method gives an example of a scheme for constructing an implicit algorithm with simple iterations for the numeric solution of the modified Cauchy problem. We formulate and prove several assertions about the computational properties of the iteration process. The effectiveness of the proposed method is proved by the numeric solution of two problems. Their numeric solutions obtained with and without parameterization of the initial problems are compared and analyzed. We consider the Lagrange problem about the propagation of sound and about a vibrating string as a test. The second example relates to modeling of the nonlinear dynamics of the deployment of a flexible rod system which freely rotates in the space relative to a fixed point.

1. Problem statement

Consider the Cauchy problem for a system of ordinary differential equations of the second order which is resolved with respect to the highest derivative

$$\ddot{u} = f(t, u, \dot{u}), \quad u(t_0) = u_0, \quad \dot{u}(t_0) = v_0. \tag{1.1}$$

Here $u(t)$ is the unknown vector-function which defines a displacement of a point in the n-dimensional Euclidean space; $t \in \mathbb{R}$ is the time; the vector-function $f = f(t, u, v)$ which implements the operator $f : \mathbb{R}^{2n+1} \to \mathbb{R}^n$ is the acceleration of the point in the space $\mathbb{R}^n$ which depends on the time, the displacement, and the velocity $v = \dot{u}$.

Mathematical modeling of many physical phenomena yields the system of equations (1.1). In particular, applied investigations in numeric modeling of dynamic processes which take place in constructions and continuums (e. g., [1]–[6]) result in equations (1.1).

Assume that the function $f \in \mathbb{C}^2(D)$, where $D \subset \mathbb{R}^{2n+1}$ is a certain domain in the Euclidean space $\mathbb{R}^{2n+1}$. Then, as is well known [7], [8], the domain D contains a unique solution of the Cauchy problem for the given initial values $y_0 = [u_0, v_0, t_0]^T \in D$. So, under these assumptions, through any point of the domain D one can draw a unique smooth integral curve $y(t) = [u(t), v(t), t]^T$

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