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MULTIVALENT FUNCTIONS FROM EXTENDED NEHARI CLASSES IN SECTORS CONTAINING A HALF-PLANE

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1. The Nehari class consists of regular in certain domain D functions $f(z)$ which satisfy the relation

$$\sup_{z\in D}[|\{f(z),z\}|R^2(D,z)] \le a(D) \tag{1}$$

where $\{f,z\} = (f''/f')' - (f''/f')^2/2$ is the Schwarzian derivative, $R(D,z)$ is a conformal radius of the domain D at point z. Let $a(D)$ be the maximal constant such that this class consists of univalent functions. According to [1], $a(D) < 2$ if D is neither a disk nor a half-plane. It is well known (see [2], [3]) that for a disk and a half-plane, $a(D) = 2$.

For a number of domains, relation (1) with an increased right-hand side cannot be used as a sufficient condition of the p-valence with $p \ge 2$. In other words, no constants $a_p = a_p(D)$ such that $a_p > a_1 = a(D)$ guarantee that any function satisfying condition (1), whose right-hand side is a_p in place of $a(D)$, is at least p-valent.

Following [4] (item 2.1), we say in this case that the Nehari functional in (1) is not p-admissible for $p \ge 2$. Let us recall that 1-admissibility of this functional is a characteristic property of quasi-circles. The following equivalence is valid for a simply connected domain D (see, for instance, [5]): $a(D) > 0 \Leftrightarrow \partial D$ is a quasi-circle.

E. Hille [3] proved in 1949 that the Nehari functional in (1) is not p-admissible for $p \ge 2$ in a disk and a half-plane. Analogous facts for the Bekker condition in the form

$$\sup_{z\in D}[|f''(z)/f'(z)|R(D,z)] \le b(D)$$

are obtained in [6] and [7].

In this paper, we extend the class of domains such that the functional in (1) is not p-admissible for $p \ge 2$. In [8], I.R. Kayumov formulates the similar theorem for infinite sectors but does not prove it correctly. Here we make certain correction of his proposition and its proof.

2. We use below the following results from [9], [10], and [1]. O. Lehto uses a quasi-conformal reflection with respect to sides of the sector $D_\varkappa = \{z : |\arg z| < \varkappa\pi/2\}$ in the form

$$\lambda(z) = \begin{cases} z^{1-1/\varkappa}\overline{z}^{1/\varkappa}e^{-i\pi}, & |\arg z| < \varkappa\pi/2; \\ z^{1-1/(2-\varkappa)}\overline{z}^{1/(2-\varkappa)}e^{-i\varkappa\pi/(2-\varkappa)}, & \varkappa\pi/2 - 2\pi < \arg z < -\varkappa\pi/2, \end{cases}$$

and thus obtains sufficient univalence conditions for two kinds of sectors. Namely, using the notation

$$\|\{f(z),z\}\|_{D\varkappa} = \sup_{z\in D\varkappa}[|\{f(z),z\}|R^2(D_\varkappa,z)],$$

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