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# EMBEDDING THEOREMS FOR ANISOTROPIC CLASSES OF DIFFERENTIABLE FUNCTIONS

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In the present article we investigate embedding theorems for classes of functions with partial derivatives belonging to various symmetric spaces. The proofs of the results described here are based on application of some interpolational constructions (see [1]), the theory of the Sobolev spaces (see [2]), and functional inequalities established in [3], [4].

In the present article we shall use terms which are customary in the theory of symmetric spaces (see [1]). If $E$ is a symmetric space (SS) of functions, then $\varphi_E$ stands for the fundamental function of the space $E$ (ibid., p. 37) and $E'$ means the space associated for $E$ (ibid., p. 65). We denote by $M_\delta$ $(0 \le \delta \le 1)$ the Marcinkiewicz space $M_\psi$ (ibid., p. 154) with $\psi(s) = s^{1-\delta}$. We assume also that the norms of the Lebesgue spaces $L_p$ $(1 \le p \le \infty)$, the Orlicz spaces $L_M$, the Marcinkiewicz spaces $M_\psi$ are defined in the standard way (see [1]). If $E_0$ and $E_1$ are SS and $E_0 \subset E_1$, then we denote by $E_1/E_0$ the space of multiplicators from $E_0$ into $E_1$, consisting of functions $h$ such that the norm

$$\|h; E_1/E_0\| = \sup\{\|hg; E_1\|, \|g; E_0\| \le 1\}$$

is finite and has a sense; the space $E_1/E_0$ is also symmetric. A collection of SS $E_1, \dots, E_k$ and constants $\tau_1 > 0, \dots, \tau_k > 0$, $\tau_1 + \cdots + \tau_k = 1$ enables us to define the space of Calderon's averages $E_1^{\tau_1} \dots E_k^{\tau_k}$ (see [1]) which is also symmetric. The notation $E \prec F$ means that the SS $F$ majorizes the SS $E$ (see [4]).

Let $\Omega$ be a domain in $R^n$ (the case $\Omega = R^n$ is not excluded), $\omega$ be a bounded open subset of the domain $\Omega$, $\square(h) = \{(x_1, \dots, x_n) : 0 < x_i < h,\ i = 1, \dots, n\}$ stand for a cube in $R^n$, and $\omega + \square(h) = \{x + y,\ x \in \omega,\ y \in \square(h)\} \subset \Omega$. To the pair $(\Omega, \omega)$ and any positive integer $\ell$ we relate the linear space $W^\ell(\Omega, \omega)$ of functions $f : \Omega \to R$ which vanish outside $\omega$ and have in $\Omega$ the Sobolev derivatives $D^\alpha f$ up to the order $\ell$, inclusively. Here and in what follows we shall use the following notation: $\alpha = (\alpha_1, \dots, \alpha_n)$ is a multisuperscript, $|\alpha| = \alpha_1 + \cdots + \alpha_n$ is the order of the multisuperscript, $\alpha\,! = \alpha_1\,! \dots \alpha_n\,!$, $D_i = \frac{\partial}{\partial x_i}$, $D^\alpha = D_1^{\alpha_1} \dots D_n^{\alpha_n}$.

Let us suppose that, for any multisuperscript $\alpha$ of the order $\ell$, we determine the maximal SS $P_\alpha(\omega)$ of functions on the set $\omega$ which are measurable with respect to the $n$-dimensional Lebesgue measure $\mathrm{mes}_n$. We denote by $W^\ell(P_\alpha; \Omega, \omega)$ the subspace of $W^\ell(\Omega, \omega)$ which consists of functions $f$ such that the norm

$$\|f; W^\ell(P_\alpha; \Omega, \omega)\| = \sum_{|\alpha| = \ell} \|D^\alpha f; P_\alpha(\omega)\|$$

makes sense and is finite. In what follows $a = \mathrm{mes}_n\,\omega$, $P_\alpha$ stands for the SS of functions on $(0, a)$, corresponding to SS $P_\alpha(\omega)$, $\tau_\gamma = (|\gamma|)!/n^{-|\gamma|}\gamma!$ where $\gamma = (\gamma_1, \dots, \gamma_n)$ is an arbitrary multisuperscript,

$$P = \prod_{|\alpha| = \ell} P_\alpha^{\tau_\alpha},$$