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Solution of a Problem of ℝ-Linear Conjugation for Confocal Elliptical Annulus in the Class of Piecewise Meromorphic Functions

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Abstract—We consider the problem of disturbance of a complex potential after insertion of a foreign inclusion in the form of a two-phase confocal elliptical annulus into a homogeneous medium. We investigate the cases of an arbitrary distribution of singularities.

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In this paper we study a mathematical model of the theory of flat heterogeneous media. This model implies the construction of a plane-parallel steady-state field $\mathbf{v}(x, y) = (v_x, v_y)$ which is potential and solenoidal in each isotropic phase of the medium under consideration. On the interface $\mathcal{L}$ of heterogeneous phases, normal (tangential) components of limit values of the vector $\mathbf{v}$ ($\rho\mathbf{v}$) are assumed to be equal. The coefficient $\rho(x, y)$, characterizing physical properties of a medium, in each isotropic component of the medium takes on a constant value. The L. M. Milne-Thomson theorem ([1], P. 153) asserts the uniqueness of a solution to this problem and gives its explicit form in the case of a single circular inclusion in the class of piecewise holomorphic functions that take on a fixed value at infinity. In papers [2–4] one generalizes the L. M. Milne-Thomson result for the case of an elliptical inclusion. In this paper we use the methods proposed in [5–7] for a further generalization of the L. M. Milne-Thomson theorem in the case of a two-phase inclusion in the form of a confocal elliptical annulus.

1. THE PROBLEM

We treat an infinite flat homogeneous medium as a plane $\mathbb{C}$ of a complex variable z. Let $f(z)$ be a complex potential with a finite number of singularities given in the plane $\mathbb{C}$. The problem consists in finding a disturbed complex potential $w(z)$ after the insertion of a foreign inclusion (in the form of a two-phase confocal elliptical annulus) into the homogeneous medium $\mathbb{C}$. Singularities of $f(z)$ may be located both inside the annulus or outside it, as well as on any of components of its boundary.

In terms of piecewise meromorphic functions $v(z) = w'(z) = v_x(x, y) - \mathrm{i}v_y(x, y)$, where $v(z)$ is complex conjugate to the velocity function $\mathbf{v}(z) = v_x(x, y) + \mathrm{i}v_y(x, y)$, the problem under consideration can be reduced to the ℝ-linear conjugation problem ([8], P. 53). Namely, let S_1, S_2, and S_3 be, respectively, an infinite medium and domains of inclusion (Fig. 1). Assume that the function $v(z) = v_k(z)$, $z \in S_k$, is meromorphic in S_k and continuous in $\overline{S}_k$ everywhere, except the corresponding singular points $f'(z)$ $(k = 1, 2, 3)$. Then limit values of $v_k(z)$ on boundaries $\mathcal{L}_1$ and $\mathcal{L}_2$ of two confocal ellipses are connected by the following boundary condition:

$$v_2(t) = A_1 v_1(t) - B_1[t'(s)]^{-2}\overline{v_1(t)}, \quad t \in \mathcal{L}_1,$$
$$v_3(t) = A_2 v_2(t) - B_2[t'(s)]^{-2}\overline{v_2(t)}, \quad t \in \mathcal{L}_2, \tag{1}$$

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