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# SKEW-SYMMETRIC ITERATION METHODS OF SOLVING STATIONARY CONVECTION–DIFFUSION PROBLEMS

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## 1. Introduction

The majority of problems of the computational mathematics can be reduced to solving systems of linear algebraic equations. To the present time a large number of algorithms for solving linear algebra problems are suggested (see [1], [2]), which mainly work effectively with matrices of special forms (three-diagonal, symmetric, banded, Toeplitz, sparse, etc.).

The convection–diffusion problems are model problems for a wide range of applied problems such as those of the hydromechanics, heat and mass exchange, motion of flying crafts. The approximation of these problems by finite differences or finite elements reduces the initial continuous problem to the necessity to solve a system of linear algebraic equations. Application of iteration methods for the solution of this problem is complicated by the fact that the corresponding systems are usually non-selfadjoint and possess a large dimension. At the same time, not always the developed apparatus of the theory of iteration methods (see [2]–[4]) can be efficiently applied to problems with a rather large non-selfadjoint part. However, systems of that sort appear as a result of the use of central-difference schemes for approximation of the convection–diffusion equation with small parameters at the higher derivative (see [5]).

Let us consider the system of linear algebraic equations

$$Au = f, \tag{1}$$

and assume that the matrix $A$ of the system is dissipative.[1] Note that for any real matrix $A$ the decomposition is valid $A = A_0 + A_1$ which decomposes the initial matrix into a symmetric $A_0 = \frac{1}{2}(A + A^*) = A_0^*$ and a skew-symmetric $A_1 = \frac{1}{2}(A - A^*) = -A_1^*$ components. Moreover, for the skew-symmetric component the representation is valid $A_1 = K_H + K_B$, where $K_H$ and $K_B$ are strictly lower and upper parts of the matrix $A_1$, respectively. Note that for these matrices the inequalities are valid $K_H = -K_B$, $K_B = -K_H^*$.

To solve system (1) we consider a one-step two-layer stationary iteration method written in canonical form in [2]

$$B\frac{y_{k+1} - y_k}{\tau} + Ay_k = f \tag{2}$$

with a certain initial vector $y_0$ and a real parameter $\tau > 0$.

[1] A matrix $A$ is said to be *dissipative* if its symmetric component is positively definite.

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