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DIRECT METHODS FOR SOLVING INTEGRAL EQUATIONS WITH LOGARITHMICALLY WEAKENED CAUCHY KERNELS ON OPEN CONTOURS

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We consider the equation

$$x(t)+\int_{-1}^{1}\frac{g(t,\tau)x(\tau)}{\tau-t}\ln^{-m}\frac{c}{|\tau-t|}d\tau=y(t),\quad t\in[-1,1],\tag{1}$$

where $g(t,\tau)$, $y(t)$ are known functions, $x(t)$ the desired function, $0<m<1$, c being a constant which will be determined in what follows.

A large number of works is devoted to solving singular and weakly singular integral equations (see, e. g., surveys in [1]–[4]). Equation (1) is, in a certain sense, intermediary between singular and weakly singular integral equations. In this article we consider collocation methods for solving equation (1). First, following [5], we investigate the properties of the corresponding singular integral. Next, on the basis of the results known in the constructive theory of functions and the general theory of approximate methods (see [6], p. 19), we carry out the substantiation of methods of collocation type and first order splines. The substantiation of these methods is realized in the generalized Hölder spaces (see [5]).

1. *On the properties of a singular integral on an open contour.* We introduce the set $\Phi=\{\varphi\}$ of positive functions φ satisfying the conditions

a) $\lim\limits_{\delta\to0+}\varphi(\delta)=0$;

b) $\varphi(\delta)$ almost increases;

c) $\sup\limits_{\sigma>0}\frac{1}{\varphi(\sigma)}\int\limits_0^\sigma\frac{\varphi(t)}{t}dt<\infty$;

d) $\sup\limits_{\sigma>0}\frac{\sigma}{\varphi(\sigma)}\int\limits_0^l\frac{\varphi(t)}{t^2}dt<\infty$, $l=\text{const}>0$.

A function $x(t)$ belongs to the set H_φ, $\varphi\in\Phi$, if

$$\omega(x,\delta)\le d_x\varphi(\delta),\quad 0<\delta<b-a,$$

where $\omega(x,\delta)$ stands for the continuity module of the function $x(t)$, $t\in[a,b]$, which is defined by the formula

$$\omega(x,\delta)=\sup_{|t-\tau|\le\delta}|x(t)-x(\tau)|,\quad 0<\delta<b-a.$$

In H_φ we introduce the norm

$$\|x\|_{H_\varphi}=\|x\|_c+H(x,\varphi),\quad H(x,\varphi)=\sup_\delta\frac{\omega(x,\delta)}{\varphi(\delta)},$$

with which H_φ turns into a Banach space.