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SUBOPTIMAL CONTROL OVER SEMILINEAR ELLIPTIC EQUATIONS WITH PHASE CONSTRAINTS. I. THE MAXIMUM PRINCIPLE FOR MINIMIZING SEQUENCES, THE NORMALITY

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1. Introduction

In the last ten years a series of works was dedicated to the problems of optimal control of distributed systems described via elliptic equations with phase constraints (see, e. g., [1]–[9]). The principal attention was paid by their authors to the proof of the Pontryagin maximum principle, the study of the connection between the stability (in a certain sense) of a problem under one-dimensional perturbation of phase constraint with the fulfillment of both the maximum principle and the regular maximum principle (see [8]).

This article differs from [1]–[9] in the following. First, in the capacity of the "basic element of the theory" we understand here not the optimal control, but a minimizing sequence (m. s.) of ordinary controls — so-called minimizing approximate solution (m. a. s.) in the Varga sense (see [10]). This allows us to consider a problem (a family of problems) with a functional parameter in the phase constraint in its most general form without additional assumptions related to the existence of an optimal control (either ordinary or generalized), without applying an extension in the sense of [10]. The necessary conditions obtained in this article for m. a. s., which we call the maximum principle for m. a. s., turn "in limit" into the ordinary Pontryagin maximum principle (in the case of existence of an optimal control). Second, we consider a perturbation of a problem, which is a perturbation of the phase constraint in the metric of the most intrinsic (infinite-dimensional) space of perturbations $C(X)$, where $X \subset \Omega$ is a compact on which the phase constraint must be fulfilled, Ω is the range of the elliptic boundary value problem. Third, in this article we discuss not only conditions for regularity of the obtained maximum principle for m. a. s., but also the conditions for its normality (the corresponding notions, see [11]–[14], are introduced in this article). Finally, fourth, here we use a method of proof (see [15]) which is principally different from that in [1]–[9]. This method consists of an approximation of the initial problem (a family of problems) with phase constraint (with a continual number of functional constraints) by a sequence of problems (a family of problems) with a finite number of functional constraints and a finite-dimensional parameter in the constraints. Afterwards, by this method, we obtain in approximating problems the maximum principle for m. a. s., and, finally, we pass to the limit as the number of approximating functional constraints tends to infinity. In such a way of approximation, on the basis of the results in [11]–[14] it turns possible to write the conditions for normality of approximating problems with a finite number of functional constraints and use them simultaneously for the proof of the normality in the initial problem with the phase constraint.

In the first part of the article we show that to the pointwise maximum principle for m. a. s. (to the ordinary pointwise maximum principle in case of the existence of an optimal control) any m. a. s.

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