Abstract thinking and bilingualism: Impact on learning mathematics

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Studies on the impact of bilingualism on academic performance in mathematics generally focus on potential problems faced by bilingual students rather than potentials and opportunities. The current research took an alternative approach and explored whether learning advantages for bilinguals may reveal themselves in solving difficult mathematical tasks that require abstract reasoning. Sixty-two bilingual and monolingual first-year students of Kazan Federal University participated in the experiment. No differences were found between bilingual and monolingual samples in solving easy and medium mathematical tasks; however, an advantage on the side of the bilingual students was observed with some of the most difficult tasks. The results from this study therefore show that bilingualism may be related to abstract thinking abilities and mathematics learning.

Keywords: Abstract thinking, learning mathematics, bilingualism.

Abstract thinking and bilingualism

Abstract thinking is a type of cognitive activity when a person moves away from specific details and begins to reason in general, using concepts, judgments and conclusions, among other structures. Moreover, abstract thinking allows us to find new things and create something new, to deviate from the rules and dogmas and to consider the phenomenon or process from different sides. The formation and development of the pupil's abstract thinking - the ability to use symbolic representation or logic – is an actual task of mathematical education.

There are many kinds of research on the relationship between mathematics and bilingualism, but only a few papers, which suggest that bilingualism may improve a person's ability to engage in more abstract or symbolic thinking processes. For example, in her work Planas (2014) argues that bilingualism can actually create opportunities for learners to deal more deeply with mathematical concepts. She observed a small sample of Catalan language learners interacting with dominant Catalan speakers while solving algebra problems in groups. The Catalan language learners used different problem-solving strategies (e.g., using a geometric approach to understand an algebraic expression) to overcome the lack of specific mathematical terminology in the academic language of instruction. The Catalan language learners also focused more on the meaning of mathematical terms than dominant Catalan speakers did because they were unfamiliar with the required terminology.

In recent years, more and more research studies have focused on the potential cognitive benefits of bilingualism. Several studies confirm that the experience of using more than one language can create unique opportunities in the bilingual brain, leading to cognitive benefits for bilinguals (Adesope, Lavin, Thomson, & Ungerleider, 2010). However, the exact nature of these benefits has proved difficult to determine in relation to specific subject contents.

On the other hand, the bilingual experience may provide other cognitive benefits. A number of researchers have supported the hypothesis that the constant switching of bilinguals from one language to another leads to an increase in executive functions (Bialystok, Craik, & Luk, 2012). Other scientists have not found any advantages for bilinguals over monolinguals in executive functioning (Paap & Greenberg, 2013). Cummins (1991) have shown that bilinguals can learn new rules more effectively than monolinguals and have an advantage in meta-linguistic awareness.

Specific research by Gentner and Goldin-Meadow (2003) confirms that knowledge representation depends on the language of instruction; hence, there may be negative consequences when knowledge is transferred from one representative system to another. In other words, negative effects are observed when the languages of learning mathematics and knowledge extraction differ. This was confirmed by Spelke and Tsivkin (2001), Campbell, Davis and Adams (2007), but also in work from our context developed by Salekhova (2016, 2018).

A bilingual student may have problems understanding the essence of a mathematical problem if he or she is not familiar with the terms used, which are associated with abstract mathematical concepts. Conversely, there may be a positive cognitive effect due to multiple transpositions. They arise whenever an abstract concept is used and transferred from one representative system to another, when there are two different language terms in the language repertoire of a person. A number of studies have shown that the linguistic complexity of texts in mathematical achievement tests can negatively affect the performance of bilinguals for whom the language of instruction and the language of testing is not their mother tongue (Haag, Heppt, Stanat, Kuhl, & Pant, 2013). However, it is unclear which specific features of language complexity contribute to this deficiency.

The present study explores whether bilingualism can potentially influence the development of abstract thinking that plays a crucial role in learning mathematics. The study's research question derives from the assumption made by the Russian psychologist Lev Vygotsky (1962). He believed that bilingualism could have positive consequences for the flexibility of human thought. He argued that the ability to express the same thought in different languages allows us to understand the symbolic function of words, to consider words in more abstract, semantic terms and to see that any particular language is only one semiotic system among many. Vygotsky studied issues concerning the relationship between bilingualism and flexibility of thinking following the awareness that bilinguals can be (dis)advantaged in traditional learning environments of Russia at that time.

Research context - bilingualism in Russia and Tatarstan

A large number of ethnic groups living within the boundaries of the Russian Federation is evidence of a complicated history of migrations, wars, and revolutions. In the process of its historical development, Russia has emerged as a multinational, multicultural and multilingual state. About 150 languages are spoken in the territory of Russia. This ethnic diversity has had a strong influence on the state education policy. The Russian Constitution guarantees all ethnic groups the right to maintain their mother tongues. Although Russian is the official language, some ethnic republics have the right to establish their own official languages besides Russian. The use of an ethnic language in education depends on the development of its written forms and literary standards.

Tatarstan is one of the ethnic republics of Russia, and Tatars constitute 53% of its population. There are two official languages – Russian and Tatar in Tatarstan. They are used for teaching almost all school subjects and the choice of the language of instruction depends on the school location (rural or urban) or the model of bilingual education (immersion, partial immersion, CLIL). Tatar is spoken by most of the people in Tatarstan as either a dominant or a second language. It plays a strong role in the construction of the Tatar cultural identity. Russian–Tatar, and Tatar–Russian bilingualism is widespread in Tatarstan if we consider bilingualism as the ability to speak two languages.

Experiment and materials

The present experiment sought to address the research question of whether bilinguals would outperform monolinguals on solving difficult symbolic math tasks. Algebra typically represents the students' first encounter with abstract mathematical reasoning and it therefore causes significant difficulties for students. Symbolic abstraction is a significant component of algebraic thinking. Algebra is a fundamental discipline in higher mathematics and plays a major role in the field of STEM education. The transition from arithmetic to algebra is a challenging task for learners, as algebraic thinking requires a shift from calculating exact values to considering relationships between quantities and operations with unknown values and variables.

One of the important topics of algebra is the study of functions, and many researchers favor teaching other algebraic topics, such as the solution of equations and inequalities based on functions. Students usually perceive functions as a tool to get answers but not as a mechanism to express the relationship between variables presented as symbolic abstractions.

Since abstract thinking represents the ability to process information, special symbolic algebraic tasks were developed in order to test for possible bilingual advantages in symbolic abstraction. Images and symbols are often used in abstract thinking, and their meaning derives solely from the thinking process. Algebraic functions are presented in an unconventional form; a new symbol is used to represent a certain sequence of basic mathematical operations. The task in our study has the following form: x&y=xy+x-y. What is 5&3? The solution of this task requires an abstract-symbolic approach and the understanding that the new symbol indicates the relationship between the variables and a certain set of mathematical operations (see Table 1).

Sample tasks			
The first task with its solution is given as an example. The answers are provided for the remaining tasks.			
1.	x∞y=xy+x-y	What is 4∞5?	
	Solution: 4∞5=4×5+4-5=19		Answer: 19
2.	x y=x/y+y-3	What is 6 2?	Answer: 2
3.	x≠y=7x-2y	What is 5≠3?	Answer: 29
4.	xαy=(3y-4x)+5	What is 1α4?	Answer: 13
Easy tasks			
1.	x@y=(x-y) + (2x+3y)	What is 4@2?	
2.	x§y=(xy) (x+y)	What is 5§3?	
3.	x€y=$(x+y)^2$ /(x-y)	What is 4€2?	
4.	x#y=x^3-y +2x+3y	What is 1#7?	
Medium tasks			
1.	x→y=(xy) (x+y)	What is (4→2) →3?	
2.	x©y=y^2 +xy	What is 5©(3©2)?	

3.	$x \leq y = xy$	$x \geq y = x/y$	What is $(9 \leq 2) \geq 3$?
4.	$[x] = x^2/2$	$\{y\} = y+2$	What is $[7]/\{7\}$?
		Difficult tasks	
1.	$<x> = 4x-2$	$2 - <x> = x-1$	What is x?
2.	$x?y = 11x/y$	$z?7 = 200$	What is z?
3.	$x \bullet y = (x-2y)/y$	$4 \bullet 5 = z \bullet 10$	What is z?
4.	$(x) = x^2 - x$	$(y) = (y-2)$	What is y?

Table 1: Symbolic mathematical tasks

Mielicki, Kacinik and Wiley (2017) introduced the types of tasks in Table 1 for school pupils. We modified them to suit our adult participants in the research. The tasks in the current study were designed to test the development of participants' abstract thinking in a way that is intended not to depend on their previous academic experiences dealing with algebraic functions.

Participants

Sixty-two bilingual and monolingual first-year students of The Institute of Philology and Intercultural Communication of Kazan Federal University (Russia) aged 18 to 20 years participated in this experiment. They perceived this kind of symbolic problems as new because they were not familiar with the concept of binary algebraic operation.

Participants were categorized as early bilinguals (N=29) and monolinguals (N=33). Bilinguals who have had a prolonged exposure to more than one language before the age of 7 are called early bilinguals. Prolonged exposure is when both parents in the family speak a language other than Russian or when the individual attended a school in which he or she did not study in Russian. We selected only early bilinguals to identify and test positive effects based on previous studies examining the cognitive benefits associated with early bilingualism (Bialystok, Craik, & Luk, 2012). Native speakers of Russian, who did not experience prolonged exposure to another language from early childhood, were considered monolinguals.

The language history of the participants was compiled with the help of interviews. In the sample of bilingual respondents, 57% indicated Tatar as their dominant language. Other dominant languages were Russian (18%), Mari (7 %), Udmurt (6%), Bashkir (4%), Tajik (3%), Uzbek (2%), Kazakh (1%), Azerbaijani (1%), and Chinese (1%).

Researchers have tried to measure dominance for many years. Some scientists use such criteria as pronunciation and vocabulary to evaluate a bilingual's languages. Others apply various tests such as naming images, recognizing words, executing a command, or translating sentences from one language to another. Based on these, the experts rate the dominant language of the participant. However, these different approaches are criticized for primitivism in the analysis of language knowledge and bilingual behavior, as the two languages are evaluated independently. We preferred an approach in which bilinguals themselves assess their level of bilingualism. We defined the dominant language as one in which the bilingual has reached an overall higher level of proficiency at a given age, and/or the language used more frequently, and in a wider range of domains.

Bilingual participants were asked to describe their level of proficiency in the dominant and the second language on a scale of 0 to 10 (0=no, 10=perfect) for "speech activity" and "understanding". Bilinguals reported similar levels of proficiency in the dominant language (M=8.36; SD=1.48) and in the second language (M=8.49; SD=1.24) for "speech activity". Similar levels were reported in the dominant (M=8.66; SD=1.34) and second languages (M=8.88; SD=1.03) for "understanding".

Russian was the dominant language of all the monolinguals. The average age of onset for second language acquisition was 7.13 (SD=1.53). Unlike bilinguals, monolinguals reported a higher level of proficiency in speaking Russian (M=9.61; SD=0.93), and a lower level in their second language (M=2.23; SD=1.16). Monolingual participants also described their level of understanding in Russian (M=9.65; SD=0.76) as higher than their second language (M=2.97; SD=1.57).

Procedures and results

Subsequently, the participants were offered to solve symbolic mathematical tasks (see Table 1): four types of symbolic math tasks of increasing complexity, consisting of four items. The solution to the first problem in each variant was given as an example. After completing each of the remaining three tasks, participants were informed whether they had answered correctly or incorrectly. Then the participants solved 12 more tasks of three levels of difficulty (easy, medium and difficult). The level of difficulty was determined by the number of new characters in the problem, as well as the number of operations required to solve the problem.

The tasks were presented in increasing complexity, and all participants performed the same tasks in the same sequence. Tasks were shown on the computer screen; the participants recorded the answers on a special worksheet. They were asked to make as many transformations and calculations as possible mentally but also had a pencil and paper, which they could use if necessary. Participants performed the tasks at their own pace and received one point for each correct answer.

Figure 1 shows the distribution of the number of correctly solved symbolic math tasks at each level of complexity by monolinguals and bilinguals. It signals minor differences between bilinguals and monolinguals in favor of the latter in solving easy and medium level problems. However, significant differences emerge in the other sense in solving symbolic math tasks of high complexity.

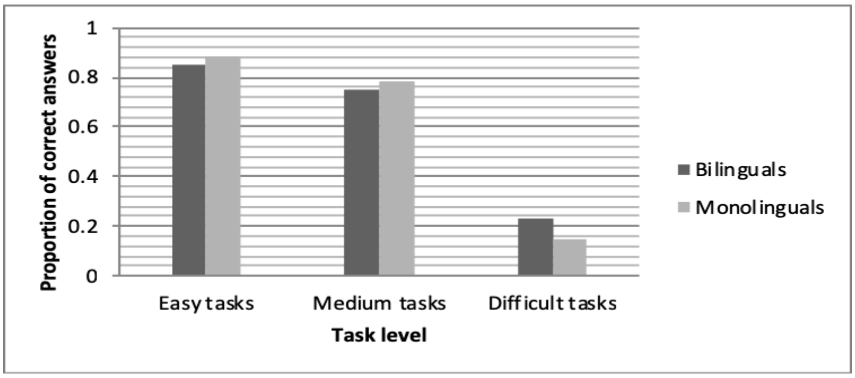

Figure 1: Performance of bilinguals and monolinguals on symbolic math tasks

A paired-samples t-test was conducted to determine the effect of bilingualism on solving symbolic math tasks. There was no significant difference between monolinguals (M=0.97; SD=0.18) and bilinguals in easier tasks (M=0.79; SD=0.41); t(60)=2, p=0.05. No evidence exists that bilingualism has an effect on solving easy symbolic math tasks that require abstract reasoning. Similarly, there was no significant difference between monolinguals (M= 0.75; SD=0.44) and bilinguals in medium tasks (M=0.83; SD=0.38); t(60)=2, p=0.05. Thus, no evidence exists that bilingualism has an effect on solving medium symbolic math tasks. However, there was a significant difference between the group of monolinguals (M=0.1; SD=0.034) and the group of bilinguals in the solution of difficult math tasks (M=0.22; SD=0.041); t(60)=2, p=0.05. Hence, we concluded that bilingualism influences the ability to solve difficult symbolic math tasks requiring advanced abstract thinking.

Bilinguals in our study performed better, particularly with the difficult problems. The advantages observed here illustrate that bilinguals may be faster to adapt to the demands of novel math tasks. Higher performance suggests that bilinguals may have some advantage in implementing the understanding of new symbolic rules in a new context. These results show that bilinguals can have superior skills in flexible selection and application of new procedures compared to monolinguals.

Discussion

Most prior research focuses on the problems encountered while learning mathematics and teaching mathematics to bilinguals and students whose home language is not the language of instruction. However, as the current study shows, bilingualism can facilitate the development of abstract thinking, which is consistent with Planas (2014). Since our sample was small, we cannot make global conclusions about the advantage of bilingualism in abstract reasoning. However, our research confirms to some extent the results obtained by Mielicki, Kacinik and Wiley (2017) for USA college students that bilinguals solve better difficult mathematical problems that require advanced abstract thinking. Our study found that bilingual students coped better with difficult abstract mathematical problems. This is how the synergetic effect of bilingualism and experience in abstract thinking manifested itself; it became obvious in those tasks that contained more ambiguity.

It is believed that executive functioning plays a crucial role in solving problems and previous studies have shown the benefits of executive functioning among bilinguals (Adesope, & Ungerleider, 2010; Bialystok, Craik, & Luk, 2012). The advantage of bilinguals in the development of executive functions could also explain the results obtained in this experiment, because memory, inhibitory control, fluid intelligence, mental flexibility and selective attention are involved in the abstract reasoning process while solving abstract math tasks.

One of the drawbacks of our investigation is the assumption that the monolingual and bilingual groups used in the study were truly comparable in every aspect, except for their linguistic skills. Paap and Greenberg (2013) expressed concern that some previously identified differences between monolinguals and bilinguals might actually be the result of differences in socioeconomic status between samples. Since socioeconomic status is known to be associated with different cognitive skills, it is important to ensure that monolingual and bilingual samples come from the same socio-economic background and share other important family characteristics (e.g., parents' education). This fact should be noted, thus an alternative explanation based on the status differences among the bilingual and monolingual participants cannot be ruled out.

Teachers and educators need to be aware of the strengths and weaknesses of bilingual students and teach them accordingly. The differences between bilinguals and monolinguals, if any, are specific to a particular task and can be quite subtle. Further research can explain our experimental results, which enrich the growing research on the complex picture of the existing effects of the bilingual experience and the strategic intellectual resource of bilingualism when learning mathematics.

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