OntoMath^{Edu} Educational Ontology: Problems of Ontological Engineering

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Abstract—This article is devoted to the tasks of ontological engineering in the field of school education. Based on the proposed approach to ontology design in the field of school mathematics, the OntoMath^{Edu} educational multilingual mathematical ontology has been developed for the school planimetry course. The features of the organization of educational ontologies that reflect the specifics of subject education in different countries, including the consideration of teaching methods and the language of study, are considered. The specific features of the organization of the subject area make it possible to take into account, among other things, the level of training of students. To solve these problems, a new structure for organizing ontologies with educational projections, educational levels, and prerequisites is proposed.

Keywords: ontological engineering, representation of mathematical knowledge, mathematical education, educational projection, prerequisites, OntoMath^{Edu}.

INTRODUCTION

Ontological engineering in the field of mathematics education is a fairly new task that requires taking into account both the specifics of the subject area and the requirements of the corresponding educational system when developing an ontology. Ontologies are currently widely used in the field of education and model various aspects of the educational process. Thus, the following areas of application of ontologies in education can be indicated: learning systems, curriculum development, competence formation, educational process management, virtual learning environments, and the dynamics (individualization) of learning processes. The most time-consuming task is, of course, the design of subject ontologies, which is used to support the functioning of learning systems and subject courses.

One of the subject ontologies in the field of school mathematics is the new OntoMath^{Edu} ontology (https://github.com/CLLKazan/OntoMathEdu). This ontology is intended to be embedded in school mathematical courses, as well as to organize educational material using modern semantic technologies.

An important task in the design of ontologies is the conjugation of ontological and linguistic models of knowledge representation. The ontological model determines the way of structuring knowledge, and the linguistic model determines the ways of functioning of language units in texts. In other words, the linguistic model determines the lexical and syntactic models of conceptual (ontological) units. The developed OntoMath^{Edu} ontology belongs to the class of linguistic (terminological) ontologies, which actually represents an ontological type knowledge base about the conceptual system and the lexical and terminological composition of the subject area; therefore, this model is successfully used for automatic processing of mathematical texts.

The OntoMath^{Edu} ontology is the central component of the developed mathematical digital educational platform, which is designed to solve problems such as (1) automatic testing of knowledge; (2) automatic recommendation of educational materials in accordance with the individual curriculum; and (3) semantic annotation of educational materials.

In its current state, the OntoMath^{Edu} ontology describes in detail the subject area of planimetry and is used for embedding into the planimetry course for a general education secondary school, which is being developed at Kazan Federal University. Since the OntoMath^{Edu} ontology is focused on educational applications, important components of the ontology are didactic relations that form an independent dimension of the ontology and act as an analogue of the *prerequisite* relation.

The developed mathematical ontology as a subject ontology is of independent interest from the point of view of classifying knowledge of a selected subject area. This circumstance allows us to formulate another important goal, namely, the presentation of the developed ontology model as the core of the Open Linked Data cloud for mathematical education. The OntoMath^{Edu} ontology was presented at CICM conferences [8, 9]. In this article, we describe the process of ontological engineering, the purpose of which is to create an educational mathematical multilingual ontology, and the main design problems and approaches to their solution.

(1) ONTOLOGICAL MODELS FOR EDUCATIONAL TASKS

Intelligent software tools usually use ontological type knowledge bases. In knowledge engineering, the term “ontology” is defined as a conceptual model of a subject area presented in a formal language that allows the use of computer tools for information processing (see, for example, [6]). At present, a number of thesauri and ontologies have been developed for a formalized representation of mathematical knowledge [11]. Separately, we note the OntoMath^{PRO} ontology developed at Kazan University [14]. This ontology contains definitions of both generally accepted mathematical concepts and the developing terminology of professional mathematics. The OntoMath^{PRO} ontology has become the basis to develop search services for mathematical texts and a recommender system for collections of documents in physics and mathematics [5]. Semantic models are being actively introduced into the practice of e-education for modeling an individual learning path, semantic annotation of educational materials, knowledge assessment, and ontological modeling of a student's cognitive profile [12, 15].

One can note the growing practice of using open linked data in education [3]. In an educational context, linked data is used to solve problems of interoperability of educational data and resources, to enrich educational content, and to personalize educational content recommendations [16].

The ontological approach has recently found application in teaching secondary school mathematics [1]. The experimental study of the authors [17] served as the basis for the assertion that the ontological approach helps pupils understand better the geometric concepts and their dynamic relationships through abstract and combined thinking.

When developing educational programs, an obligatory step is to determine the dependences of training modules and the sequence of their study [12]. Semantic dependences between learning modules are set using the prerequisites of the educational subject considered. In educational ontologies, the didactic *prerequisite* relation connects ontology concepts based on the learning standards of the corresponding education system [10].

(2) DESIGNING THE ONTOMATH^{EDU} MULTILINGUAL MATHEMATICAL ONTOLOGY

Let us describe in detail the main problems and methods for solving them when designing a multilingual mathematical ontology intended for a secondary school planimetry course.

The construction of a multilingual educational ontological model of the subject area of planimetry required formalization of the concepts of the content of the planimetry course and the relationships between them and the construction of full-fledged alternative language systems, taking into account the organization of the educational process. For this purpose, it was necessary to solve the following tasks:

- Forming a database of educational materials in various languages;
- Extracting geometric concepts from educational materials;
- Establishing correspondence of ontology terms in different languages in different learning systems;
- Building hierarchies of geometric concepts;
- Building relationships between the concepts of the subject area;
- Introducing specific relationships that reflect the process of teaching the chosen subject of school mathematics into the ontology;
- Describing the concepts and all their relationships (definitions, images, links to external resources, etc.).

When designing the OntoMath^{Edu} ontology, the planimetry course studied at schools of the Russian Federation in grades 7–9 was chosen as the first course of school mathematics; in the Republic of Tatarstan this course is taught in a number of schools in the Tatar language. In the United Kingdom, a similar course has been designed for secondary school students (Secondary Education).

The development of the OntoMath^{Edu} ontology aims to create a multilingual ontology with terminological systems of mathematical concepts in Russian, English, and Tatar. The terminological base of the ontology for the planimetry course in Russian and Tatar is based on the methodological systems for teaching planimetry that have been adopted in the Russian Federation. At the same time, the Tatar terminology in ontology is mainly translated equivalents of terms in Russian. The relationships between the Russian and English terminological parts of the ontology required the development of a separate model of their interrelations. The proposed solutions for building such a model of interrelations make it possible to create other multilingual educational subject ontologies in the future.

Selection of concepts. Geometric concepts were selected using the expert knowledge of the authors of the article, as well as educational and reference materials in the Russian, English, and Tatar languages. We can specify the geometry textbook by L.S. Atanasyan et al.; additional textbooks on elementary geometry by G.K. Gordin et al.; the geometry textbook by L.S. Atanasyan in the Tatar language (translated from Russian by Z.Kh. Bilalova and V.Z. Zakirov); Russian--Tatar and Russian--Tatar--English explanatory dictionaries and reference books of mathematical terms (I.G. Galyautdinov, L.I. Galieva, L.L. Salekhova, et al.); the Russian--Tatar and Tatar--Russian dictionaries Lingvo 12.0, etc.; geometry textbooks in English such as H. Africk (2013), D.C. Alexander and G.M. Koeberlein (2016), A. Ameen, K. Khan, and B. Padmaja Rani (2012), J. Cummins, J.A. Carter, G.J. Cuevas, R. Day, and C. Malloy (2012), etc.; and the Russian--English dictionaries “Multitran” and Lingvo 12.0, etc.

The choice of the language and the system of teaching planimetry required taking into account the following factors in the ontology model:

– Methodological features of the process of teaching planimetry in the education systems of the Russian Federation and the United Kingdom, namely the presence of geometric concepts in the planimetry course for schoolchildren; the sequence of studying concepts and the established relationships between them; the proposed definitions of geometric concepts, etc.

– The educational orientation of the ontology requires identification of the levels of pupils mastering the subject (at each educational level of the study of planimetry, its own set of geometric concepts and relationships between them is offered).

– Linguistic features of Russian and English when translating mathematical terminology, including the choice of the term translation option used when there are several options (for example, vysota/height, altitude; krug/circle, disk) or the problem of the lack of a term in the school curriculum (kite/deltoid, complementary angle/angle that is complementary to an angle of 90 degrees), etc.

Description of concepts. An important task in constructing a description of a concept in an ontology is the selection of definitions of a given concept from various sources. Existing school textbooks often provide different definitions of individual geometric concepts. For example, below are the definitions of the concept of a polygon in the main geometry textbooks in Russian.

(1) A simple closed broken line (A.V. Pogorelov).

(2) A flat polygon or polygonal area is the final part of the plane, which is bounded by a polygon (A.V. Pogorelov).

(3) A figure composed of segments AB, BC, ..., FA, so that adjacent segments do not lie on one straight line, and nonadjacent segments do not have common points (L.S. Atanasyan).

(4) A figure consisting of the sides of a polygon and its inner region (L.S. Atanasyan).

(5) A closed broken line that does not have self-intersections limits a polygon (I.F. Sharygin).

(6) A figure formed by a simple closed broken line and the part of the plane bounded by it (I.M. Smirnova).

The above definitions refer the concept of a *polygon* to fundamentally different classes (a polygon as part of a plane, or a polygon as a broken line, or a combination of a broken line and a part of a plane). The choice of definition makes it possible to establish the position of a geometric concept in taxonomy and subsequently to build numerous relationships between the concept under consideration and other concepts (for example, area and perimeter, belonging/not belonging of points to a polygon, etc.). In particular, for a polygon, our choice is related to the definition that relates a polygon to a part of a plane, since the area of a polyline cannot be measured.

Ontology projections. To highlight the system of terms that define the area of concepts of the planimetry course of a particular education system, projections of the OntoMath^{Edu} ontology (hereinafter referred to as ontology projection) were created. Ontology projection is a system of ontology concepts and relations connecting them, which is a formalized model of the content of school mathematics in a certain education system.

The OntoMath^{Edu} ontology presents two ontology projections: the first in Russian in the education system of the Russian Federation and the second in English in the education system of the United Kingdom. Ontology projections are connected with each other through common concepts of geometric notions that are present simultaneously in the two systems of teaching planimetry. Each ontology concept has a name in Russian and English. Educational levels are additionally marked in the ontology projection. The educational level of the OntoMath^{Edu} ontology includes the set of concepts of the subject area that are studied at a certain level of education in accordance with a given educational system. To mark the belonging of an ontology concept to an educational level, the “Educational level” property is introduced.

In the ontology projection in Russian, the following values of the “Educational level” property are distinguished: grade 7, grade 8, grade 9, profile grade 8, profile grade 9, additional level. In the ontology projection in English, the values are Key Stage 1, Key Stage 2, Key Stage 3, and Key Stage 4.

Each subsequent educational level includes the set of concepts of the lower levels.

Thus, the combination of concepts with the properties of the educational level grade 7, grade 8, and profile grade 8 makes it possible to identify the concepts that correspond to the level of training of a pupil who has finished studying at profile mathematical grade 8. Thus, the selected educational levels are modeled in terms of inclusion.

The selection of projections and educational levels in the ontology is a new step in the design of multilingual educational ontologies.

(3) STRUCTURE OF THE ONTOMATH^{EDU} ONTOLOGY

Based on the proposed approach to ontology design in the field of school mathematics, the OntoMath^{Edu} educational multilingual mathematical ontology has been developed for the secondary school planimetry course.
Formally, the OntoMath^{Edu} ontology is represented as a tuple $O=<C, Cr, Rel, Hc, Hrel, A>$, where C is a set of classes, Cr is a set of materialized relationships (subclass C), $Rel \subset C \times C$ is a set of class properties, Hc: $C \times C$ is a class hierarchy, $Hrel$: $R \times R$ is a relationship hierarchy, A is a set of axioms over classes and relationship constraints.
Ontology projection on the educational system is
$O^*=<C^*, Rel^*, Hc^*, Hrel, A^*>$,
where C is a set of classes, Cr is a set of materialized relationships (subclass C), $Rel <= C \times C$ is a set of class properties, Hc: C×C is a class hierarchy, $Hrel$: R×R is a relationship hierarchy, and A is a set of axioms over classes and relationship constraints.
The structure of the OntoMath^{Edu} ontology includes the following layers of representation, which are further called levels: linguistic level, subject ontology level, and upper ontology level.
The top-level ontology level contains the UFO (Unified Foundational Ontology) ontology as a representation model [7].
The level of subject ontology contains the concepts of the subject area of school mathematics. Concepts are linked to external resources from the Linked Open Data (LOD) cloud, such as the DBpedia project (https://www.dbpedia.org) and the ScienceWISE and OntoMath^{PRO} ontologies. In addition, based on the MMT resource naming scheme (Module system for Mathematical Theories) [13], concepts can be associated with the MitM ontology [4] and, through it, with the concepts of various computer algebra systems.
The subject ontology level was designed in accordance with the following modeling principles:
(1) Conceptual model of school mathematics. OntoMath^{Edu} reflects the set of concepts of school mathematics that is used in educational mathematical literature.
(2) Strict adherence to ontological differences defined in the top-level ontology. For example, we explicitly divide concepts into types and into roles.
(3) Materialization of subject relationships. Many-place mathematical relationships in the ontology are presented as concepts rather than as object properties. Due to this, mathematical relationships are entities of the first order and can act as objects of statements.
(4) Multilinguality. Ontology concepts contain names in English, Russian, and Tatar.
The structure of the ontology includes the following: hierarchy of objects, hierarchy of materialized relationships, network of points of view. The current version of OntoMath^{Edu} contains 916 concepts, while the hierarchy of objects includes 675 classes, and the hierarchy of materialized relationships includes 212 classes. 1280 “class-subclass” relationships were introduced into the ontology.
Concept classes have names in English, Russian, and Tatar. In accordance with the requirements of the top-level ontology, the objects of the subject ontology are divided into concept-types and concepts-roles.
The top level of the type hierarchy includes the concepts *Figure on a plane*, *Axiom of planimetry*, *Theorem of planimetry*, *Problem of planimetry*, *Unit of measurement*, and *Tool for measuring or building geometric figures*.
The ontology defines the following relationships represented by object and annotation properties and their subproperties: (1) the “have an argument” relationship, (2) the ontological dependence relationship, (3) the “Part--whole” relationship, (4) the objectivity relationship, (5) the “Prerequisite” relationship and the “Educational level” relationship, and (6) the “External resource” relationship.
The ontology contains a hierarchy of materialized relationships. Materialized relationships serve as a means to represent the statements of a formal mathematical theory based on first-order predicate logic as an RDF graph.
A formal theory statement of the form $R(c_1, \ldots, c_n)$ is represented in terms of ontology as a general scheme of RDF triplets (Fig. 1):

Fig. 1.
Thus, relationships between concepts are first-order entities and can act as the object of statements. An example of a statement of the form $R(c_1, \dots, c_n)$ is the statement *tangent*(*a*,*b*,*c*), i.e., "line *a* touches circle *b* at point *c*."
The upper level of the hierarchy of materialized relationships contains the concepts-classes *Mutual arrangement of geometric figures on a plane*, *Relationship of comparison of geometric figures*, *Plane transformations*, and *Metric property of a geometric figure*.
A network of points of view. In addition to universal statements about mathematical concepts, the ontology contains statements tied to individual points of view. Points of view are modeled using the "Descriptions and Situations" design pattern and based on the DOLCE+DnS Ultralite top-level ontology [2].
Currently, the following types of points of view are introduced into the ontology:
(1) Definitions. The same concept may have different definitions according to different points of view.
(2) Educational levels. There are different educational levels in the ontology, which refer to different educational systems.
External links. The OntoMath^{Edu} ontology is currently connected with the DBpedia resource. 142 connections with this resource were built semi-automatically and checked manually based on the approach proposed in [9].
Prerequisites. To use an ontology for educational purposes, it is necessary to establish logical connections between concepts that determine the order of studying concepts in a real educational process according to the corresponding national curriculum. Concept A is called a prerequisite for concept B if the study of concept A is required to understand concept B. For example, understanding the concept *Addition* is required to understand the concept *Multiplication*.
An example of a chain of prerequisites is the sequence of ontology concepts is *Point* [7th grade] → *Ray* [7th grade] → *Angle* [7th grade] → *Right angle* [7th grade] → *Right triangle* [7th grade] → *The Pythagorean Theorem* [8th grade].
OntoMath^{Edu} offers two approaches to defining prerequisites: directly by linking two concepts and indirectly by organizing concepts by educational levels.
The linguistic level of the OntoMath^{Edu} ontology consists of multilingual dictionaries (lexicons) that define the ways of expressing concepts from the subject ontology in Russian, English, and Tatar. Each ontology concept has a name (word or phrase), the meaning of which corresponds to this concept. Each concept is also supplied with a set of text inputs, i.e., language expressions reflecting the meaning of the concept, which are extracted from mathematical texts by special algorithms.
To represent multilingual ontology lexicons, a meta-ontological model has been developed, which is based on the existing ontologies OntoLex/Lemon (https://www.w3.org/community/ontolex/; https://www.w3.org/2016/05/ontolex/), LexInfo (https://lexinfo.net/), and PreMOn (Predicate Model for Ontologies, https://premon.fbk.eu/). Thanks to this, the developed lexicons can be integrated into the cloud of Linguistic Linked Open Data (LLOD, http://linguistic-lod.org/llod-cloud). The developed meta-ontological model also contains a superstructure over the indicated ontologies, with the help of which syntactic frames from lexicons can be associated with materialized relationships.
The design of the OntoMath^{Edu} ontology was done in the Protégé editor.

CONCLUSIONS
This article presents the process of developing the new OntoMath^{Edu} educational ontology and briefly describes its current version. It identifies the main problems of ontological engineering in the educational sphere, which are related to the specifics of subject education in different countries in different educational systems, including those that require taking into account teaching methods and the language of study. The OntoMath^{Edu} ontology is the first educational

multilingual general-purpose mathematical ontology presented in the Linked Open Data (LOD) cloud.

The OntoMath^{Edu} ontology is in the process of further development, and in the near future it is planned to apply the modeling principles tested in this project to the development of other sections of school mathematics.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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FIGURE CAPTIONS

Fig. 1. Model of a materialized relationship in OntoMath^{Edu}.
map(R) - отображение имени предиката R на URI материализованного отношения из OntoMath^{Edu} --> *map(R)* is the mapping of the name of the predicate R to the URI of the materialized relation from OntoMath^{Edu};
map (c_i) - отображения имен констант аргументов предиката на URI объектов из OntoMath^{Edu} --> *map* (c_i) is the mapping of the names of predicate argument constants to object URIs from OntoMath^{Edu}.

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