PARAMETERS OF NUMERICAL AND ANALYTICAL EPHEMERIDES OF THE MOON USE COMPLEX SYSTEMS ANALYSIS METHODS

PhD Student Arthur Zagidullin[1]
Sci. Assoc. PhD Vladimir Usanin[1]
Assoc. Prof. PhD Natalia Petrova[1,2]
Prof. Dr. Yury Nefedyev[1]
PhD Student Alexey Andreev[1,2]
[1] Kazan Federal University, Institute of Physics, **Russia**
[2] Kazan State Power Engineering University, **Russia**

ABSTRACT

This work considers the study of libration dynamics of the planets' moons. To analysis such processes, it is necessary to use methods for investigating complex systems, as an influence from neighboring celestial bodies on spin-orbital motion of a moon should be taken into account. This fully applies to the Moon's rotation about its axis. The construction of modern theories of the physical libration of the Moon (PLM) is based on gravimetric, seismic, satellite observations, and lunar laser ranging, which in its turn has allowed to increase the accuracy by several orders of magnitude compared to the PLM theories of the end of the 20th century. At the moment, some of the most accurate ephemerides are DE created at JPL NASA [1]. The DE ephemerides are based on the coefficients of the Chebyshev polynomials produced by integrating motion equations of the Solar system bodies; the physical libration of the Moon as well as the Earth's nutation are taken into account in the integrating process [2]. To produce PLM parameters from DE ephemerides, one should use a complex algorithm of determining necessary data [3]. It must be noted that the study of the orbital - rotational motion of the moon in the section of celestial mechanics is allocated in a separate area. Such a separation is primarily connected with the fact that this problem cannot be solved within the framework of the 2-body problem. It is necessary to take into account the second perturbing body, it follows from this that the first integrals of motion will not be enough to obtain an exact analytical solution to this problem (before the advent of computers, it was necessary to pre-calculate the coordinates of the moon, most important for sailors of those times).

Keywords: numerical and analytical ephemerides of the Moon lunar physical libration, numerical modeling

INTRODUCTION

Based on the application of planetary ephemerides, it is possible to take into account subtle effects in the lunar rotational dynamics and implement highly accurate reduction of modern satellite and ground-based observations. With a purpose of further development of the libration theory, in this work a comparative analysis of the author's

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semiempirical theory of PLM [4] with the data produced from planetary ephemerides is conducted. In the standard coordinate system, libration angles are determined depending on the time scale. A possibility of the application of the results produced in this work is considered: 1) while building Hamiltonian and corresponding differential equations with taking into account the introduction of additional potentials considering viscoelastic properties of the lunar body and its double layer structure [5]; 2) for bringing the Moon's rotation equation to the form adapted for highly accurate integration providing the solution accuracy of 0.1 ms over time interval longer than 3 years [6].

Many scientists were engaged in the solution of 2-body problem, these works made it possible to create first semi-analytical series of solutions that are not resistant to changes in the Stokes coefficients (responsible for the distribution of masses inside the body of the moon), and then to analytical series that allow you to vary the Stokes coefficients without resorting to recalculating huge series decisions. Other features in the differences in solutions in our work are not significant, therefore we are not talking about them. With the advent of computers, the procedure for solving differential equations of rotation was facilitated, but with the help of these numerical methods we lose the possibility of flexible variation of the results, therefore, even now analytically calculations are actively used. The interpretation of the now numerical ephemeris (pre-calculation of the coordinates of the stars) DE for the libration of the moon requires analytical support to the methods of numerical integration. Williams [1], one of the authors of DE, actively uses analytical support for all the results of numerical integration of the equations of the physical libration of the moon, which is natural, since only with the help of the analytical apparatus can we identify the physics of each harmonic, in particular, to separate the forced libration from free libration.

TRANSFORMATION OF THE REFERENCE FRAME

The first step is the transformation coordinate system of dynamical theory. Because the dynamical theory is constructed in equatorial non-rotating coordinate system, so the first step is a transformation into ecliptic coordinate system. The second step is a transformation into rotating coordinate system. Transformation from equatorial coordinate system into ecliptic coordinate system of epoch J200 is carried out on fixed angle ε_0 = 84381.448 arc seconds:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{ecliptic} = R_x(\varepsilon_0) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{equator} \tag{1}$$

The mean longitude of Earth relative the Moon is determined by

$$L_{sideric} = F + \Omega - p_A - 180°, \tag{2}$$

where F is the polynomial Delaney's argument on time (mean argument of latitude the Moon), p_A is the general precession in longitude.

The second stage is the transformation of the coordinate frame of. Precession from planets that changes the Earth's orbit relatively the Sun is not considered in the theory [7]. Therefore, at the beginning we do not know whether the base plane is close to the ecliptic of the current date or to the ecliptic of a fixed epoch. Analysis of orbital theory [8] showed that if we will take the ecliptic epoch J2000 as base plane, then on the interval of 800 years the discrepancies reach about 400 arc second, that is caused by the angle between of planes the ecliptic of date and ecliptic of epoch J2000.

RESULTS OF COMPARISON AFTER ALL TRANSFORMATIONS

After bringing the theories under consideration to the unified system of coordinates, we can talk about the results of their comparison.

Fig. 1 gives the differences in longitudes and latitudes after taking into account the secular acceleration, without taking precession into account from the planets. Fig. 2 shows the discrepancies in longitude and latitude after joint consideration of the precession from the planets and secular acceleration.

Table 1 lists all the numerical values of the series used to convert the reference frame. These series were taken from [8] pp. 67-68, except for the last two arguments, which were taken from [9].

Unfortunately, there is ambiguity in the values of some arguments. The contribution of these quantities leads to a discrepancy by an amount not exceeding 2 arc seconds.

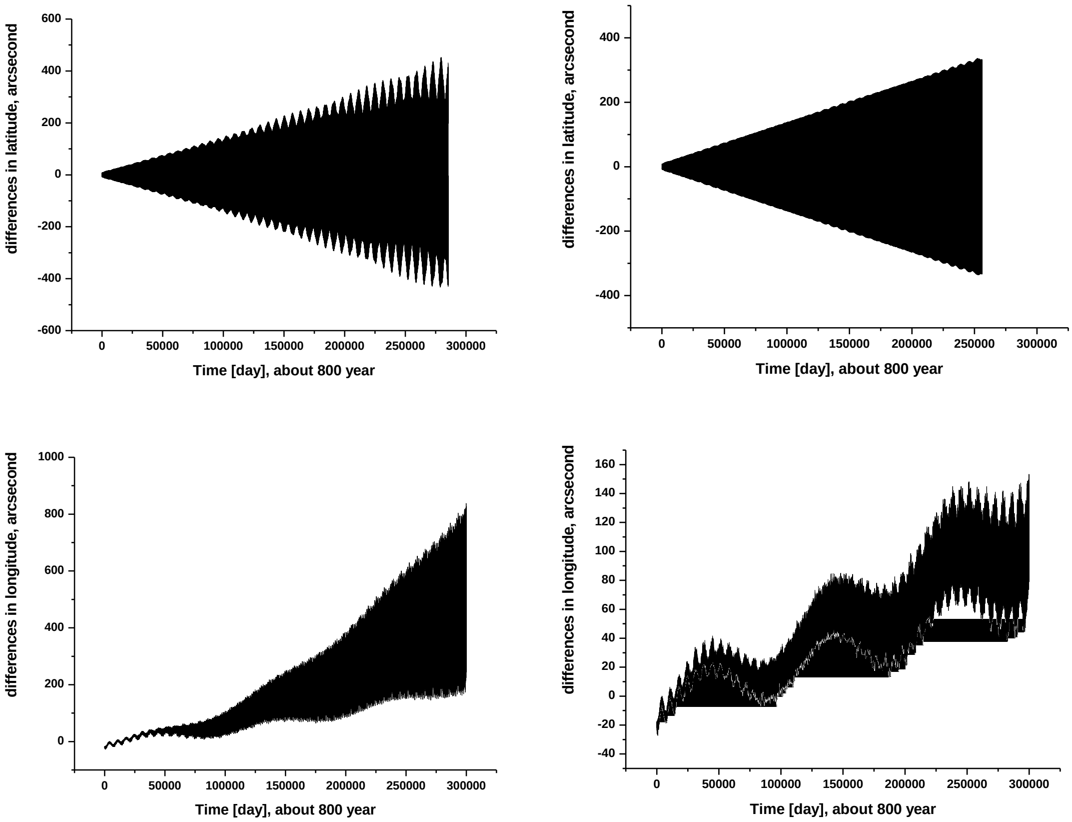

Figure 1 Effect of secular acceleration: on the left without acceleration, on the right - after accounting the acceleration. First row is difference in latitude, the second row is difference in longitude

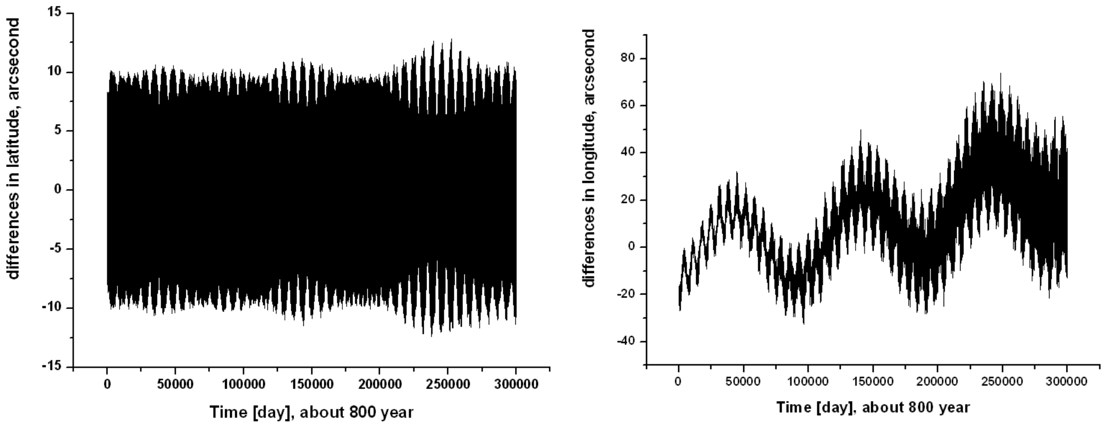

Figure 2 Combined effect of secular acceleration and precession from planets. On the left - differences in latitude, on the right - in longitude.

In this work, we considered the numerical ephemeris of DE, in the context of storage and use of the ephemeris, in relation to obtaining the orbit of the Moon relative to the Earth. The ephemeris is based on the coefficients of the Chebyshev polynomial obtained by integrating the equations of motion of objects in the solar system, which also include the physical libration of the moon and the nutation of the Earth. To obtain the libration components from ephemeris, it is necessary to consider a more complex algorithm for data extraction.

Table 1 Numerical values of parameters under used

Designation	t^0, °	t^1, ″	t^2, ″	t^3, ″	t^4, ″
L	134.96340251	1717915923.2178	31.8792	0.051635	-0.00024470
l'	357.52910918	129596581.0481	-0.5532	+0.000136	-0.00001149
F	93.27209062	1739527262.8478	-12.7512	-0.001037	0.00000417
D	297.85019547	1602961601.2090	-6.3706	0.006593	-0.00003169
Ω	125.04455501	-6962890.5431	7.4722	0.007702	-0.00005939
p_A (radian)	0	0.02438175	$5.38691 \cdot 10^{-6}$	0	0
Π_A	174°52′34″.982	-869.8089	0.03536	0	0
π_A	0	47.0029	-0.03302	0.00006	0

CONCLUSION

At the Kazan University, the lunar studies are traditional. The development of the theory of physical libration is one of the directions for studying our satellite. Petrova [4] constructed the analytical theory of the physical librations of the Moon (LPhL) with an accuracy of 0.01 arc seconds based on the analytical theory of the orbital motion of the Moon of Gutzwiller, Schmidt [7]. Now, we are developing a numerical approach in the theory of LPhL and, as the first stage, we constructed a numerical copy of the analytic theory. This numerical theory [3] is constructed within the framework of the main problem, considering the orbital and rotational motion independent, using the same analytical theory [2] for orbital motion description. The problems arose when we began to go over to the numerical ephemeris DE432 to describe the motion of the centre of mass. The biggest discrepancy with the theory [2], more than 200 arcseconds, has arise in the longitude already at the beginning of the integration, and reached more 2000 seconds over a year because of the revealed secular trend with an angular coefficient of 4.6 seconds. The reason for this behaviour of residual differences could be both an incorrect transition to dynamical ephemerides, and the influence of those effects that were not taken into account in theory [2]. It is obvious that the physical libration of the Moon is very sensitive to the orbital theory used. That is why the analysis of orbital theories is necessary, firstly, in order to ensurethe correctness of the application of the theory, and secondly, to evaluate the influence on the rotation of the Moon of those subtle effects that distinguish modern theories from the main problem model. When passing from analytical theory to numerical dynamical ephemerid, one must understand the specifics of the construction of the theory. The understanding of all the subtleties in orbital theories is an integral part of obtaining an exact theory of the rotation of the Moon

The work was focused on the analysis of the numerical theory of DE432. We performed a study on the accuracy, disturbances and structure of the DE432 [10]. The theory of physical libration was brought to the J2000 era. When bringing the system to the J2000 era, planetary precession was taken into account. An analysis was also made of the motion of the lunar center of mass for 800 years. In carrying out the work, it was found that if you do not take into account nonlinear terms in the expansion of physical libration, then for 800 years the secular acceleration will be 830 arcsec. As a result, the residual differences will be about 15 arcsec in latitude and 80 arcsec in longitude. These differences are available in the article on the diagrams. Such differences of the analytical theory from the numerical one arise due to the disregard for disturbances from the planets in the analytical theory of physical libration [11].

The obtained results and advantages of using of LPhL when developing navigation support for the lunar missions are: 1) capacities on the control of spacecrafts and prediction of their physical and technical parameters over long time interval are extended [12]; 2) the possibility of checking adaptation and stability for LPhL of hybrid systems for spacecraft is considered [13]; 3) the creation of local systems for the work in complex environments remote from the Earth; 4) the recognition and use of local conditions without assistance; 5) for achieving the high level of space missions safety [14]. The results produced will be applied for determining selenographic parameters for lunar bases and also for preparing and implementing lunar space missions [15]. The creation of stationary lunar bases is a promising direction not only for robotic

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exploration of the Moon but also as a launch pad for preparing manned space flights to Mars.

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