Pulse shaping by a frequency filtering of a sawtooth phase-modulated cw laser

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The spectrum of a cw field the phase of which experiences a periodic sawtooth modulation is analyzed. Two types of the sawtooth phase modulation are considered. One is created by combining many harmonics of the fundamental frequency. The second is produced by an electro-optic modulator fed by the relaxation oscillator, which generates a voltage slowly rising during charging of the energy storage device and dropping fast due to discharge by a short circuit. It is proposed to filter out the main spectral component of the sawtooth phase-modulated field. This filtering produces short pulses from the phase-modulated cw field. The duty cycle of this train is equal to the modulation period, while duration of the pulses depends on the rate of the phase drop. In the case of harmonics, this duration is $4N+2$ shorter than the period of the fundamental frequency, where N is the number of the harmonics. In the case of the charge and discharge, duration of the pulses is close to $T_D/2$, where T_D is a drop time of the phase in the discharge period. Depending on the modulation frequency, the proposed method is capable to produce pulses with duration ranging from nanoseconds to a fraction of a picosecond.

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I. INTRODUCTION

Ultrafast optics is applied in widespread domains including but not limited to high-field laser matter interactions, ultrafast time-resolved spectroscopy, high-precision frequency metrology, and development of optical clocks, nonlinear microscopy, and optical communications (see, for example, Refs. [1,2]). Initially, short periodic pulses are generated by high-repetition-rate mode-locked lasers. However, in this method the generated pulses and their corresponding spectral lines can suffer from instability problems. Alternative passive systems generating pedestal-free optical pulses with high peak power from a low-power laser employ a large variety of methods to compress the pulses. Among them one can mention, for example, acousto-optic modulators [3–5], frequency chirping followed by dispersive compensators [6–10], dispersive modulators [11,12], high rf power spatial modulation of the field phase by electro-optic modulators (EOMs) followed by a lens [13], and modification of phases and amplitudes of the spectral components of the phase-modulated field by programmable filters to engineer the desired spectrum of the field [14]. Passive systems using phase-modulated cw lasers offer several advantages. Among them are lower cost and complexity, easy tuning of the frequency comb offset, continuous tunability of the duty cycle, and reasonably stable operation without active control.

Phase modulating techniques, mentioned above, are based on the phase manipulation of the spectral components of the frequency comb, which leads to phasing of these components. The number of the components with noticeable amplitudes increases with increasing of the modulation index and hence the bandwidth of the comb increases. Therefore, duration of the compressed pulses by the phasing of the frequency comb components shortens with increasing of the phase modulation index. The duty cycle of the pulse train is always equal to the modulation period.

A different method of pulse compression was recently reported in Refs. [15–18]. The capabilities of this method were experimentally demonstrated for gamma photons with long duration of a single-photon wave packet [15,16]. Splitting of a single-photon long pulse into short pulses can be used to create time-bin qubits, the concept of which was introduced before in quantum informatics for optical photons [19,20].

The method [15] is also based on harmonic phase modulation of the radiation field. However, instead of subsequent control of the spectral components producing their phasing, absorption (removal) of the component with “the wrong phase” is proposed. This removal leads to phasing of the remaining spectral components. The removal method is also flexible and allows fine control of the duration and repetition rate of the pulses.

An appreciable shortening of the pulses in the method [16,17] is achieved for high phase-modulation index as in the previous passive methods of the pulse shaping. However, with increase of the modulation index and removal of “the wrong component” the number of which increases with the increase of the modulation index, the generated pulses are grouped into bunches. The number of pulses in each bunch is equal to the number of the removed spectral component. Duration of the individual pulse in the bunch is $4n$ times shorter than the phase modulation period, where n is the number of the removed spectral component. A drawback of the pulse shortening with the increase of n is the accompanying decrease of the pulse amplitude and increase of the pedestal, i.e., the field amplitude in the “dark windows” between bunches. Therefore, the most favorable case in the removal method is deletion of the $n=1$ or -1 component. In this case the pulse duration is four times shorter than the phase modulation period. To overcome the drawback, arising for high modulation index, several spectral components are proposed to be removed in Ref. [17]. In this case, one needs specially prepared filters complicating the method.

In this paper, a modification of the phase modulation technique with subsequent removal of one of the spectral components of the comb is proposed. In this variant, appreciable pulse compression can be achieved for a moderate value of the modulation index, which is even smaller than that produced by an EOM fed by the half-wave voltage V_π, which makes a π shift of the field phase.

The core idea of the method is a sawtooth phase modulation in which the phase periodically ramps upward and then sharply drops. It is proposed to construct a linear phase rise and sharp drop using additive synthesis of many harmonics of frequency Ω with decreasing amplitudes according to the law $1/n$, where n is the number of the harmonic $n\Omega$. The larger the number N of the highest harmonic, which is $N\Omega$, the sharper the phase drop. This simple model of the phase modulation allows one to describe analytically all the details of the pulse shaping based on one spectral component removal in the spectrum of the phase-modulated field. Duration of the compressed pulse, predicted by the model, is $2(2N+1)$ times shorter than the period $T = 2\pi/\Omega$ of the phase modulation.

Sawtooth phase modulation produces from a single line cw field a frequency comb with a period Ω. The main spectral component of the comb with $n = 1$ has a frequency offset $-\Omega$ with respect to the carrier of the cw field, ω_r. The phases of the satellites of the main component $n_m = 1$ have unusual properties very different from those produced by harmonic phase modulation [17]. The nearest satellites with numbers $n_m \pm 1$ have the same phase as the main component. The phase of the next pair, $n_m \pm 2$, is shifted by π. The components $n_m \pm 3$ are again in phase with the main component, etc. Therefore, simple removal of the main component results in phasing of the satellites each time when $\Omega t = (2k+1)\pi$, where k is an integer. Thus, to generate pulses from the sawtooth phase modulation cw field there is no need to manipulate with phases of the spectral components of the comb. Duration of the pulses shortens appreciably for a large number of harmonics.

The physical processes in the pulse generation from the combs, produced by harmonic and sawtooth phase modulations, are quite different. However, it is demonstrated in this paper that both can be theoretically described by the same quite simple method. It can be shown that this method can be easily applied to describe, for example, experiments [21] with a many-pixel liquid crystal modulator (LCM) array controlling phases of the frequency comb, produced by harmonic phase modulation.

Understanding of the physics of the proposed method allows one to extend the method to the case of nonideal sawtooth phase modulation with periodic nonlinear phase rise and exponential phase drop. The faster the phase drops, the shorter the pulse is produced.

It is proposed to implement the filtering of the main component of the comb by a cloud of cold atoms, atomic vapors, organic molecules doped in a polymer matrix, and a liquid crystal phase and amplitude modulator. Depending on the value of the modulation frequency and selected frequency filter, one can generate a sequence of pulses ranging from nanoseconds to a fraction of a picosecond.

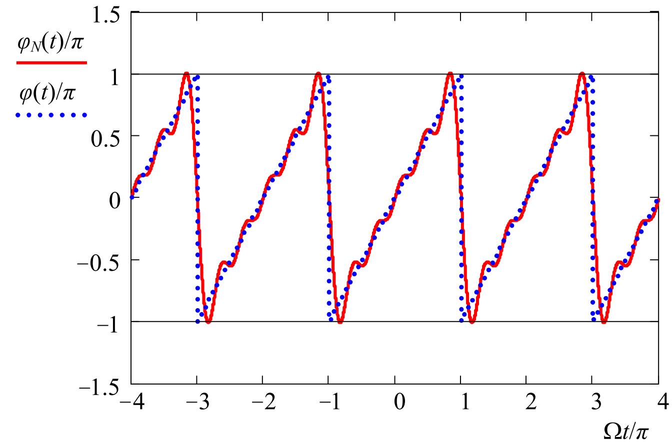

FIG. 1. Time evolution of the sawtooth phase $\varphi(t)$ (blue dotted line) and the phase $\varphi_N(t)$, synthesized from $N = 5$ harmonics (solid red line). Both are normalized to π. Horizontal black bars show limiting values of the phase change, $\pm\pi$.

The paper is organized as follows. In Sec. II, a sawtooth phase modulation I, created by mixing many harmonics of the fundamental frequency, is considered. In Sec. III, frequency filtering of the phase-modulated field is discussed. Comparison of the sawtooth phase modulation I with harmonic modulation is presented in Sec. IV. In Sec. V, periodic sawtooth phase modulation, which consists of a slowly rising stage according to the law $(1 - e^{-t/T_R})$ and a fast dropping stage according to e^{-t/T_D}, and frequency filtering of the phase-modulated field are considered. Frequency filtering methods are discussed in Sec. VI.

II. SAWTOOTH PHASE MODULATION I

We consider cw radiation field $E(t) = E_0 \exp(-i\omega_r t + ikz)$, which after passing through the electro-optic modulator acquires a sawtooth phase modulation:

$$E_{\rm EO}(t) = E(t)e^{i\varphi(t)}, \tag{1}$$

where

$$\varphi(t) = \sum_{n=-\infty}^{+\infty} (\Omega t - 2\pi n)\left\{\theta\left[t - T\left(n - \frac{1}{2}\right)\right] - \theta\left[t - T\left(n + \frac{1}{2}\right)\right]\right\}, \tag{2}$$

Ω and $T = 2\pi/\Omega$ are the modulation frequency and period, n is an integer varying from $-\infty$ to $+\infty$, and $\theta(x)$ is the Heaviside step function. This kind of phase modulation is shown in Fig. 1 by the dotted blue line. Here, in the definition of the cw radiation field, Eq. (1), only the positive frequency part of the field is considered for convenience.

Physically, it is difficult to make an instantaneous phase drop after a linear ramp up. This problem can be solved by using additive synthesis of many harmonics of frequency Ω with decreasing amplitudes (see, for example, Refs. [22,23]). Fourier transforms,

$$\frac{1}{T}\int_{-T/2}^{T/2} \varphi(t)e^{-in\Omega t}\,dt = i\frac{(-1)^n}{n}, \tag{3}$$

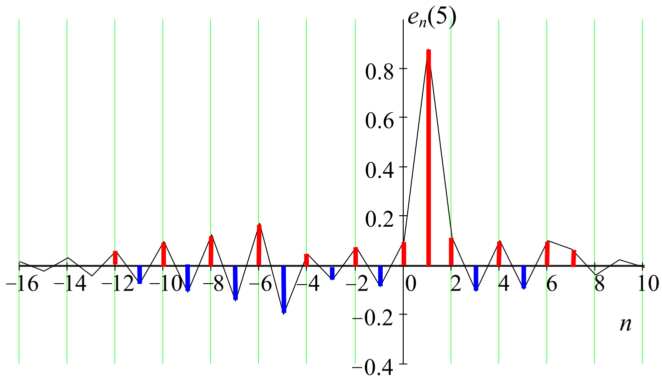

FIG. 2. Frequency content of the field with sawtooth phase modulation synthesized from $N = 5$ harmonics. The amplitudes of the spectral components are shown by red (positive amplitudes) and blue (negative amplitudes) bars. Their maxima are linked by black solid lines for visualization.

for $n \neq 0$ and $\int_{-T/2}^{T/2} \varphi(t)dt = 0$ for $n = 0$, give the frequency content of $\varphi(t)$ for the limited number N of the harmonics, i.e.,

$$\varphi_N(t) = 2\sum_{n=1}^{N}(-1)^{n+1}\frac{\sin(n\Omega t)}{n}, \tag{4}$$

where N defines the highest frequency $N\Omega$ of the Fourier content of the synthesized periodic phase evolution. An example of $\varphi_N(t)$ with $N = 5$ is shown in Fig. 1 by the red solid line. It is interesting to notice that the modulation index of the main spectral component with $n = 1$ is 2, while together with the other four components the maximum phase shift is π, which can be produced by the half-wave voltage V_π applied to the EOM.

Fourier transform

$$\frac{1}{T}\int_{-T/2}^{T/2} e^{i\varphi_N(t)-in\Omega t}dt = e_n(N) \tag{5}$$

allows one to find the Fourier content of the field $E_{\rm EO}(t) = E(t)e^{i\varphi_N(t)}$ transmitted through the EOM, which can be expressed as

$$E_{\rm EO}(t) = E_0 e^{-i\omega_r t + ikz}\sum_{n=-\infty}^{n=\infty} e_n(N)e^{in\Omega t}. \tag{6}$$

The amplitudes of the spectral components $e_n(N)$ are shown in Fig. 2. The component with $n = 1$ has the largest amplitude. For example, for $N = 5$ this amplitude is $e_1(5) = 0.879$. With increase of N, this amplitude tends to 1, i.e., $e_1(10) = 0.936$ for $N = 10$ and $e_1(50) = 0.987$ for $N = 50$. Numerical analysis shows that $e_1(N)$ can be approximated as $e_1(N) \approx 1 - 1.33/(2N + 1)$.

The satellites of the component with $n = 1$ have smaller amplitudes, and their signs change such that the nearest components to the main one, which we denote as $n_m = 1$, are positive ($n_m \pm 1$), the next pair ($n_m \pm 2$) is negative, then the next amplitudes with numbers $n_m \pm 3$ are positive, etc., until $n_m \pm N$ (see Fig. 2). The amplitudes of the components in each pair are not equal, i.e., $e_0(5) = 0.102$ and $e_2(5) = 0.111$ for the nearest pair and $e_{-1}(5) = -0.085$ and $e_3(5) = -0.105$ for the next pair. With increase of N, the absolute values of the amplitudes of the satellites decrease, while the number of satellites with noticeable value of the amplitudes increases, resulting in the spectrum broadening of the field. In addition, the amplitudes of the pairs with numbers $n_m \pm 1$ and $n_m \pm 2$ tend to be equal for large N, i.e., $e_2(50) = e_0(50) = 0.013$ and $e_3(50) = e_{-1}(50) = -0.013$. This tendency is almost conserved for the satellites with large numbers. For example, we have $e_{10}(50) = e_{12}(50) = 0.012$ and $e_{-8}(50) = e_{-10}(50) = 0.011$.

III. FREQUENCY FILTERING OF THE PHASE-MODULATED FIELD I

If we selectively remove the main component n_m of the phase-modulated field without change of all other spectral components, we expect that the remaining $2N$ components will phase and rephase with the period T. To explain this point we just consider, for example, the interference of only the two nearest spectral pairs of the main component,

$$E_{tp}(t) = E(t)[e_{-1}(N)e^{-i\Omega t} + e_0(N) + e_2(N)e^{2i\Omega t} + e_3(N)e^{3i\Omega t}], \tag{7}$$

and assume that $e_0(N) = e_2(N) = -e_{-1}(N) = -e_3(N) = a$, which is the case when $N = 50$ and $a = 0.013$. Then, Eq. (7) can be expressed as

$$E_{tp}(t) = 2aE(t)e^{i\Omega t}(\cos\Omega t - \cos 2\Omega t). \tag{8}$$

This equation shows that when $\Omega t = 2k\pi$ the amplitude $E_{tp}(t)$ is zero because of destructive interference of the spectral pair $e_0(N)$ and $e_2(N)$ with the pair $e_{-1}(N)$ and $e_3(N)$. When $\Omega t = (2k + 1)\pi$ the amplitude $E_{tp}(t)$ is equal to $-4aE_0$ due to constructive interference of the spectral pairs. Here, k is an integer.

It should be noted that the amplitudes of the harmonics $(n_m + n)\Omega$ and $(n_m - n)\Omega$, produced by the sawtooth phase modulation, have always the same sign for $n \leqslant N$. For $n > N$ their signs become opposite (see Fig. 2). Therefore, only $2N$ spectral components interfere destructively or constructively depending on the value of Ωt.

Analysis of the interference of all the pairs of the frequency comb, which is described by Eq. (6) with the removed main component n_m, is quite complicated. Essential simplification is achieved if one employs the method proposed in Refs. [16,17]. It is suggested to express the filtered comb as

$$E_f(t) = E(t)\left(\sum_{n=-\infty}^{n=\infty} e_n(N)e^{in\Omega t} - e_{n_m}(N)e^{in_m\Omega t}\right) \tag{9}$$

or

$$E_f(t) = E(t)[e^{i\varphi_N(t)} - e_{n_m}(N)e^{in_m\Omega t}]. \tag{10}$$

Then, the intensity of the filtered field $I_f(t) = |E_f(t)|^2$ can be presented as

$$I_f(t) = I_0\{1 - 2e_{n_m}(N)\cos[n_m\Omega t - \varphi_N(t)] + e_{n_m}^2(N)\}, \tag{11}$$

where $I_0 = E_0^2$. According to this expression the evolution of the phase $\psi(t) = n_m\Omega t - \varphi_N(t)$ fully defines the interference

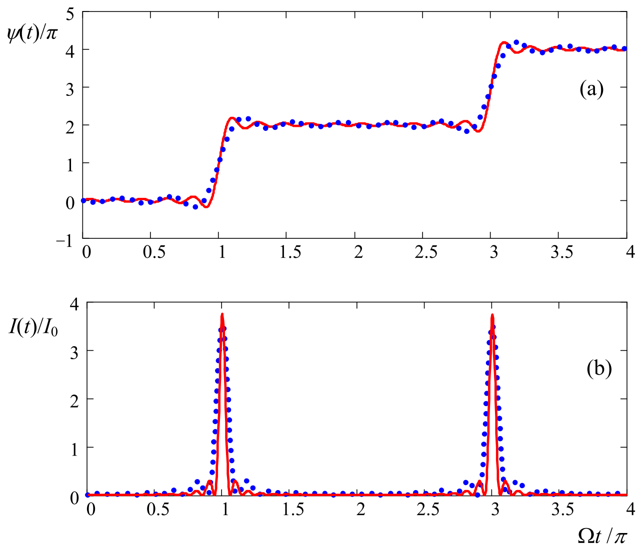

FIG. 3. (a) Time evolution of the phase $\psi(t) = n_m\Omega t - \varphi_N(t)$, which specifies the interference of the comb with the scattered field $E_{sc}(t)$. (b) Time dependence of the intensity of the pulses normalized to I_0. Solid red lines correspond to the sawtooth phase modulation with $N = 10$, and blue dots correspond to $N = 5$.

of the spectral pairs, mentioned above. Formally, this interference can be considered as an interference of the whole comb, $E_{EO}(t)$, with the removed component the phase of which is changed by π, i.e., with the field $E_{sc}(t) = -e_{n_m}(N)E(t)e^{in_m\Omega t}$. In the case of atomic or molecular filters, the field $E_{sc}(t)$ has a physical meaning. This field is coherently scattered in the forward direction by atoms the resonant frequency of which is $\omega_r - \Omega$ [16,17,25,26].

The scattered field $E_{sc}(t)$ is in phase with the field $E_{EO}(t)$ when $\psi(t) = (2k+1)\pi$, where k is an integer. Constructive interference of these fields produces a pulse with intensity $I_{max} = I_0[1 + e_{n_c}(N)]^2$. When $\psi(t) = 2k\pi$, destructive interference of the fields results in the drop of intensity to the level $I_{min} = I_0[1 - e_{n_m}(N)]^2$. Substantial contrast between the pulse maximum and the pedestal is achieved if $e_{n_m}(N) \longrightarrow 1$. For example, for the sawtooth phase modulation, synthesized from five harmonics, we have $e_1(5) = 0.879$, which gives $I_{max} = 3.53I_0$ and $I_{min} = 0.015I_0$. Thus, for $N = 5$, there is a 23.7-dB contrast ratio between the pedestal and the pulse maximum. For $N = 10$ and 50, the contrast ratios are 30 and 43.5 dB, respectively.

Time evolution of the phase difference $\psi(t)$ of the comb $E_{EO}(t)$ and coherently scattered field $E_{sc}(t)$ is shown in Fig. 3(a) for $N = 5$ and 10. In time intervals $(k + 1/2)T < t < (k + 3/2)T$, the phase difference $\psi(t)$ is close to $2\pi(k + 1)$, which results in destructive interference of the fields. Here k is an integer. On the borders of these time intervals the phase difference $\psi(t)$ jumps from $2\pi(k + 1)$ to $2\pi(k + 2)$ crossing the value $2\pi(k + 3/2)$. At the crossing when $\psi(t) = 2\pi(k + 3/2)$, the pulse is formed due to constructive interference of the fields [see Fig. 3(b)]. The larger the number of harmonics N, the faster phase $\psi(t)$ crosses the value $2\pi(k + 3/2)$ and the shorter the pulse that is formed. The slope of the phase change at the crossing point, which takes place at $t_k = (k + 3/2)T$, is equal to $\partial\psi(t)/\partial t\mid_{t_k} = (2N + 1)\Omega$. Thus, the rate of the crossing point is proportional to the modulation frequency Ω and the number of harmonics N constituting the sawtooth phase modulation.

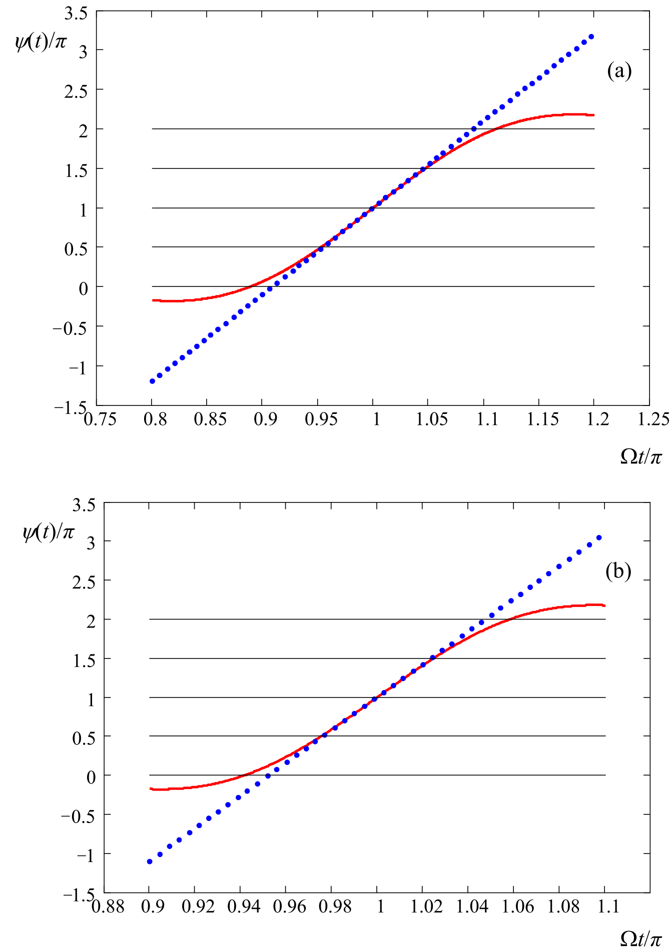

FIG. 4. Time evolution of the phase $\psi(t)$ near one of the crossing points, i.e., at t_k with $k = -1$, is shown by the red solid line. Its approximation by linear time dependence is shown by blue dots (see the text for details). The number of harmonics constituting the sawtooth phase modulation is $N = 5$ (a) and 10 (b).

The phase rise at the crossing point t_k can be approximated as a linear function $\psi(t) \approx \Omega t_k + (2N + 1)\Omega(t - t_k)$ with the slope $\partial\psi(t)/\partial t\mid_{t_k} = (2N + 1)\Omega$. This function fits well with the evolution of the phase $\psi(t)$ around the crossing points (see Fig. 4, where the t_{-1} crossing is shown). In the case $k = -1$, shown in the figure, the linear approximation is reduced to $\psi(t) \approx (2N + 1)\Omega t - 2\pi N$. It is predicted that at $t_{-1} = T/2$ we have $\Omega t_{-1} = \pi$ and $\psi(t_{-1}) = \pi$. Then, constructive interference produces a field with maximum intensity $I_{max} = I_0[1 + e_{n_m}(N)]^2$, which is $I_{max} = 4I_0$ if $e_{n_m}(N) \longrightarrow 1$. Neglecting the pedestal $I_{min} = I_0[1 - e_{n_m}(N)]^2$, one can roughly estimate from Fig. 4 that intensity of the pulse drops to its half value $I_{max}/2$ when $t = t_{-1} \pm t_{half}$ where t_{half} satisfies the relation $(2N + 1)\Omega t_{half} = \pi/2$. At times $t = t_{-1} - t_{half}$ and $t_{-1} + t_{half}$ the phase $\psi(t)$ takes the values $\pi/2$ and $3\pi/2$, respectively. Intensity of the pulse drops two times at these moments since $\cos\psi(t)$ in Eq. (11) is zero. Thus, the full width at half maximum (FWHM) of the pulse can be estimated as $2t_{\rm half} = T/(4N+2)$, i.e., it is $(4N+2)$ times shorter than the period of the phase modulation. Taking into account that $e_{n_c}(N)$ is slightly smaller than 1, we numerically found that $t_{\rm half}$ satisfies slightly a different relation, which is $(2N+1)\Omega t_{\rm half} = \pi/1.923$. It does not deviate significantly from our rough estimation.

IV. COMPARISON OF THE SAWTOOTH PHASE MODULATION I WITH HARMONIC MODULATION

Harmonic phase modulation $\varphi_h(t) = M \sin \Omega t$, where M is the modulation index, creates a frequency comb:

$$E_{\rm EO}(t) = E(t) \sum_{n=-\infty}^{+\infty} J_n(M) e^{in\Omega t}, \tag{12}$$

where $J_n(M)$ is the Bessel function of the nth order. The amplitudes of the harmonics $+n\Omega$ and $-n\Omega$ have the same sign if n is even, and they have opposite signs if n is odd.

It is well known that any periodic phase modulation is incapable to produce periodic intensity oscillation. This statement simply follows from the relation $E(t)e^{i\phi(t)}E^*(t)e^{-i\phi(t)} = I_0$, where $\phi(t)$ is an arbitrarily time varying phase. In the case of harmonic phase modulation, one can calculate directly the product $\sum_{n=-\infty}^{n=+\infty}\sum_{k=-\infty}^{k=+\infty} J_n(M)J_k(M)e^{i(n-k)\Omega t}$ and find that it equals 1 and does not contain harmonic oscillation [see, for example, Eqs. (11)–(15) in Ref. [24]]. This is because of a particular balance between the amplitudes and phases of the harmonics. It was shown in Ref. [24] that any changes (even small) in the amplitudes or phases of the spectral components in Eq. (12) result in intensity oscillations. This effect was proposed to use for high-resolution spectroscopy in Ref. [24].

In Refs. [16,17] it was shown that removal of the nth component of the comb, Eq. (12), creates pronounced pulses if the amplitude of this component, $\sim|J_n(M)|$, has a global maximum. For $n = 1$ this takes place when $M = 1.8$ and $J_1(1.8) = 0.582$. Pulses with maximum intensity $I_{\rm max} = 2.5I_0$ are formed when $\Omega t = (2k+1)\pi$, where k is an integer. Minimum intensity of the radiation field, $I_{\rm min} = 0.175I_0$, takes place at the centers of the dark windows when $\Omega t = 2k\pi$. Therefore, the contrast ratio between the pulse maximum and the pedestal is 11.5 dB, which is 2 (2.6) times smaller compared with the pulses created by the filtering of the sawtooth phase-modulated field with $N = 5$ (10). Duration of the pulses is equal to a quarter of the phase modulation period T, while for the sawtooth phase-modulated field with $N = 5$ (10) this duration is 22 (42) times shorter than T. Thus, substantial pulse shortening is obvious in the case of the sawtooth phase-modulated field.

If, in the case of the harmonic phase modulation, one removes the nth spectral component with $n > 1$, then the cw field is transformed into bunches of pulses separated by the dark windows. The number of pulses in the bunch is equal to n. Duration of the individual pulse in the bunch is $4n$ times shorter than T. For example, for $n = 3$ pulse duration is $T/12$. The spectral component with $n = 3$ has global maximum $J_3(M) = 0.434$ when $M = 4.2$. Thus, pulse shortening takes place when the modulation index is properly increased. At the same time, the contrast ratio $I_{\rm max}/I_{\rm min}$ decreases with increase of n. For example, for $n = 3$, this contrast drops down to 8 dB since $I_{\rm max}$ decreases to $2I_0$ and $I_{\rm min}$ increases to $0.32I_0$.

Moderate performance of the harmonic phase-modulated field originates from the incomplete phasing of the spectral components after removal of the selected component of the comb. For example, for $n = 1$, the intensity of the filtered field is

$$I_f(t) = I_0\left|\sum_{k=1}^{+\infty} J_{1-k}(1.8)e^{-ik\Omega t} + J_{1+k}(1.8)e^{ik\Omega t}\right|^2, \tag{13}$$

the main components of which are $0.34e^{-i\Omega t} + 0.306e^{i\Omega t}$, $-0.582e^{-2i\Omega t} + 0.1e^{2i\Omega t}$, $0.306e^{-3i\Omega t} + 0.023e^{3i\Omega t}$, and $-0.1e^{-4i\Omega t} + 0.004e^{4i\Omega t}$. Large difference of the amplitudes of the interfering components, for example, -0.582 and 0.1 for $k = 2$, does not give appreciable pulse shortening in spite of the large number of the interfering components (four pairs with noticeable amplitudes for $n = 1$).

Thus, filtering of the sawtooth phase-modulated cw field produces shorter pulses with larger contrast between the pulse maximum and the pedestal than filtering of the harmonic phase-modulated field. Frequency synthesis of many harmonics is a routine method now in modern electronics. For example, a cw synthesized microwave generator is capable to produce a field with a frequency of several GHz and spectral width below several Hz [27]. One can also find examples of experimental implementation of the sawtooth wave generation, for example, in Refs. [22,23].

The method of LCM control of phases of the comb components, created by the harmonic phase modulation of the cw, shows even better performance. However, LCM technique suffers from fiber-to-fiber insertion loss, circulator loss, focusing back into the fiber mode after the pulse shaper, etc., which result in 11.6-dB total loss (see Ref. [21]).

LCM is capable to change selectively the phases of the spectral components of the comb, Eq. (12). Thorough theoretical analysis of phasing of spectral components of the harmonic phase-modulated field by LCM is not yet available. Below, to explain briefly the physics of this technique we consider, as an example, the case when $M = 5$. Among the components with noticeable amplitudes, five are negative, i.e., their phases are π shifted with respect to others. Their amplitudes are proportional to $J_{-7}(5) = -0.053$, $J_{-5}(5) = -0.261$, $J_{-3}(5) = -0.365$, $J_0(5) = -0.178$, and $J_1(5) = -0.328$, where the numbers of the components coincide with the orders of the corresponding Bessel functions. If the phases of all the necessary components are properly adjusted, the modified field $E_{\rm mod}(t)$ can be expressed as follows:

$$E_{\rm mod}(t) = E(t)\left[|J_0(M)| + 2\sum_{n=1}^{+\infty}|J_n(M)|\cos n\Omega t\right]. \tag{14}$$

The spectral components of the modified frequency comb are phased when $\Omega t = 2k\pi$, where k is an integer. Dependence of the maximum intensity of the pulses $I_{\rm max}$ on the modulation index M is shown in Fig. 5(a). This dependence can be roughly approximated as $I_{\rm max} = (2M+1)I_0$. When $\Omega t = (2k+1)\pi$, the spectral components interfere destructively and the radiation intensity drops to the minimum value $I_{\rm min}$.

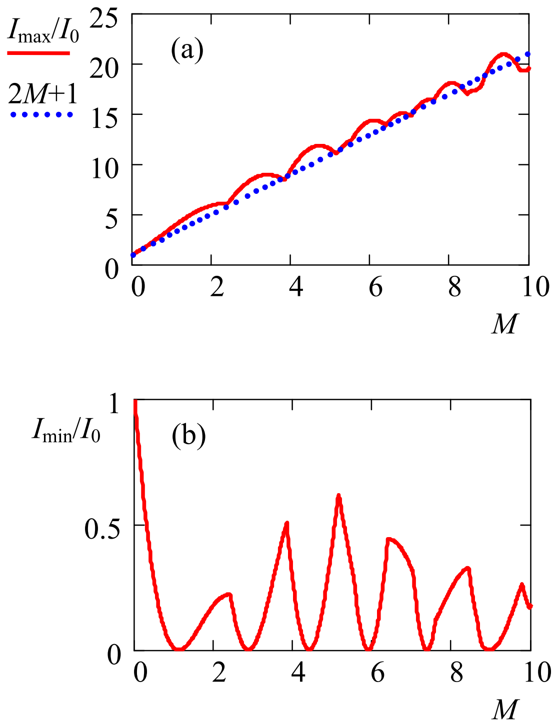

FIG. 5. (a) Dependence of the maximum intensity of the pulse I_{max} on the modulation index is shown by the red solid line and its approximation is shown by blue dots. (b) Dependence of the minimum intensity in the dark windows on M.

Its dependence on M is shown in Fig. 5(b) demonstrating noticeable oscillations. From this figure it follows that to keep large contrast between the pulse maximum and the pedestal special care is needed when choosing an appropriate value of the modulation index. For example, for $M = 5$ the contrast ratio is only 14 dB, which is even smaller than that which can be achieved by the sawtooth phase modulated field with $N = 5$.

Numerical calculation of the pulse durations for M equal to 5, 10, and 15 shows that duration of the pulses (FWHM) can be roughly approximated as $T/\pi M$. For example, for $M = 5$ pulse duration is nearly 15 times shorter than the modulation period. This result is comparable with the performance of the sawtooth phase-modulated field with $N = 5$.

Interference of the comb with the scattered field, proposed to describe pulse shaping in Refs. [16,17], is also applicable for the description of LCM controlling the harmonic phase-modulated field. This is because in a real experiment the phases of only a limited number of harmonics are modified. Therefore, instead of Eq. (14), one has to use the equation

$$E_{mod}(t) = E(t)[e^{i\varphi_h(t)} + e_{comp}], \tag{15}$$

where e_{comp} is the sum of the modified components the amplitudes E_n of which are changed as $-2E_n$, i.e., their amplitudes are doubled and phases are shifted by π. This formal procedure results in the appropriate change of phases of the selected spectral components. In the case of the programmable filters [14], the field $e_{comp}E(t)$ is virtual.

The core idea of the method is a consideration of the interference of the comb with a fictitious field or fields essentially simplifying analysis. This is very similar to the method of images in electrostatics, which helps to find the electric field created by the real charge on the conducting surface. The problem is essentially simplified by introducing the image charge with opposite sign located mirrorlike symmetrically in the bulk with respect to the conducting surface and thus simulating the conducting medium.

V. SAWTOOTH PHASE MODULATION II

The sawtooth phase modulation can be also realized by an EOM fed by a sawtooth voltage, which is produced by relaxation oscillators. In the simplest type of this oscillator the energy storage capacitor is charged slowly but discharged rapidly by a short circuit through the switching device. Then, the ramp voltage can be described by the equation

$$U_R(t) = U_0(1 - e^{-t/T_R}) + U_{min}, \tag{16}$$

where U_0 is a maximum charge voltage, T_R is a rise time, and U_{min} is an initial voltage, from which the ramp starts. The voltage drop is described by

$$U_D(t) = U_{max}e^{-t/T_D}, \tag{17}$$

where U_{max} is a voltage when the discharge starts and T_D is a drop time.

A periodic phase modulation, produced by such a sawtooth voltage, can be expressed as follows:

$$\varphi_{RO}(t) = C\sum_{k=0}^{+\infty}\phi[t - k(T_R + T_D)], \tag{18}$$

where

$$\phi(t) = \phi_R(t) + \phi_D(t), \tag{19}$$

$$\phi_R(t) = (1 - e^{-t/T_R})[\theta(t) - \theta(t - T_R)], \tag{20}$$

$$\phi_D(t) = [e^{-(t-T_R)/T_D} - e^{-1}][\theta(t - T_R) - \theta(t - T_R - T_D)]. \tag{21}$$

Here, for simplicity, it is assumed that the rise and drop time periods are equal to T_R and T_D, respectively, and the time independent part of the phase is disregarded, resulting in the condition $\phi(0) = 0$. The maximum value of the phase $\phi(t)$ at $t = T_R$ is taken equal to 2π, which gives $C = 2\pi/(1 - e^{-t/T_R})$. The period of this sawtooth-phase modulation is $T = T_R + T_D$. Time evolution of the phase $\varphi_{RO}(t)$ is shown in Fig. 6. The case when the rise and drop time periods are not equal to T_R and T_D, respectively, will be considered separately.

Fourier transform

$$\frac{1}{T}\int_0^T e^{i\varphi_{RO}(t) - in\Omega t}dt = c_n \tag{22}$$

allows one to find the Fourier content of the field $E_{RO}(t) = E(t)e^{i\varphi_{RO}(t)}$, which is

$$E_{RO}(t) = E_0e^{-i\omega_r t + ikz}\sum_{n=-\infty}^{n=\infty} c_n e^{in\Omega t}, \tag{23}$$

where $\Omega = 2\pi/T$.

Following the derivation method presented in Sec. III, one can obtain that removing the frequency component $\omega_r - \Omega$,

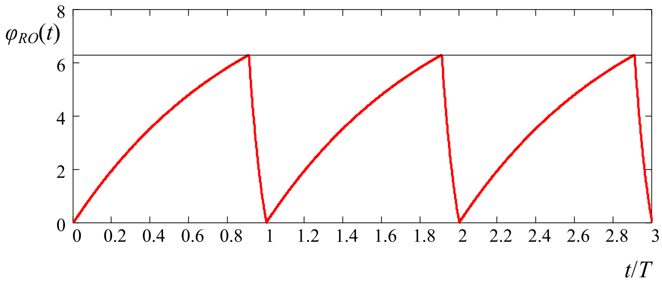

FIG. 6. Time evolution of the phase $\varphi_{RO}(t)$. The time scale is normalized to $T = T_R + T_D$. The parameters of the sawtooth phase modulation are related as $T_D = T_R/10$. The black horizontal bar corresponds to 2π.

the amplitude of which is c_1E_0, modifies the phase-modulated field as

$$E_f(t) = E(t)(e^{i\varphi_{RO}(t)} - c_1 e^{i\Omega t}), \tag{24}$$

the intensity of which is

$$I_f(t) = I_0\big[1 - 2a_1 \cos\psi_{RO}(t) + a_1^2\big], \tag{25}$$

where

$$\psi_{RO}(t) = \Omega t + \xi_1 - \varphi_{RO}(t). \tag{26}$$

Here a_1 and ξ_1 are the modulus and argument (phase) of the complex number c_1, i.e., $c_1 = a_1 \exp(i\xi_1)$.

An example of the formation of pulses is shown in Fig. 7(a) by the blue dotted line for the case when the phase drop is ten times faster than the phase rise, i.e., for $T_D = T_R/10$. In this case $a_1 = 0.892$ and $\xi_1 = 0.808 \approx \pi/3.9$. Absolute value a_1 of c_1 is close to unity. Therefore the peak pulse intensity is 3.58 times larger than the intensity of the cw field I_0. Evolution of the phase $\psi_{RO}(t)$, which governs the interference of the incident field, $E_{\mathrm{EO}}(t)$, with scattered field, $E_{\mathrm{sc}}(t) = -c_1E(t)e^{i\Omega t}$, is shown in Fig. 7(a) by the red solid line. Each time when $\psi_{RO}(t)$ crosses the value $(2k+1)\pi$, the pulse is formed.

A zoom in on the area of the pulse formation around $t = T$ is shown in Fig. 7(b). Numerical analysis gives an estimation of the pulse duration t_p (full width at half maximum), which is $0.041T_R$ for $T_D = T_R/10$, i.e., $0.037T$. Thus, during a short time of the phase drop T_D, the pulse is mainly formed within the time interval $0.41T_D$, which is slightly less than a half of T_D.

Slight asymmetry of the pulses and small kinks at their right shoulders originate from two factors. The first comes from the unbalanced competition of the phases Ωt and $\varphi_{RO}(t)$ in the function $\psi_{RO}(t)$ and the second results from the nonlinear time dependence of the phase $\varphi_{RO}(t)$ in the rising and dropping stages.

Unbalanced competition of the phases follows from the definition of $\varphi_{RO}(t)$. The phase Ωt equals 2π at $t = T$, while $\varphi_{RO}(t)$ reaches this value earlier at time $t = T_R$. Nonlinear time dependence of $\varphi_{RO}(t)$ in the rising and dropping stages produces nonzero phase $\xi_1 \approx \pi/3.9$ of the main spectral component. Both factors result in the rise of the phase $\psi_{RO}(t)$ beyond $2\pi k$ at the end of the phase rising stage producing a kink in the pulse intensity at $t = kT$ when the phase $\psi_{RO}(t)$ starts to reduce due to the unbalanced competition of phases Ωt and $\varphi_{RO}(t)$ [see Fig. 7(b)].

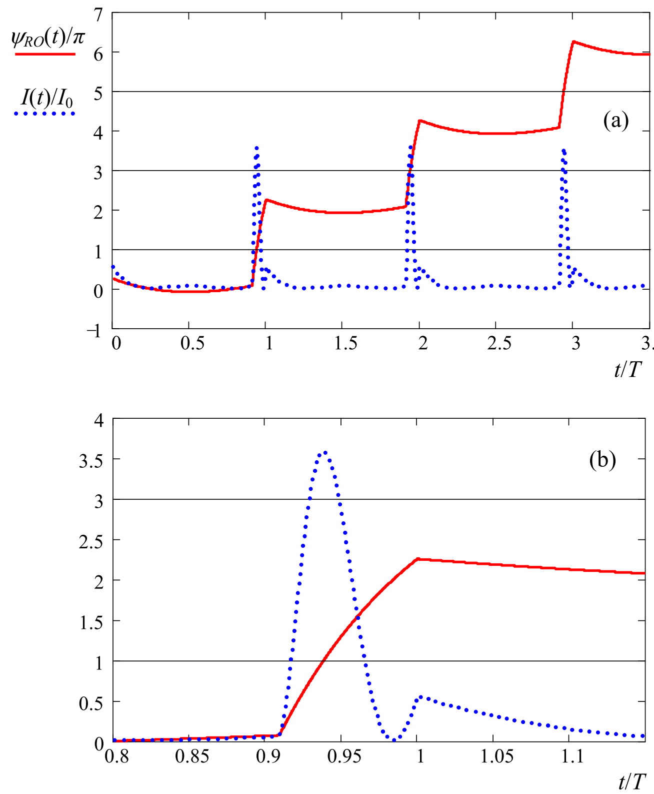

FIG. 7. (a) Time evolution of the phase $\psi_{RO}(t)$ (in units of π) is shown by the red solid line. The sequence of pulses, generated by the frequency filtering, is shown by the blue dotted line. The field intensity is normalized to I_0. The time scale is in units of the period T. Both plots correspond to the case when $T_D = T_R/10$. (b) Zoom in on the area of the pulse formation around $t = T$. In both plots, horizontal black thin lines indicate the levels corresponding to the phase values equal to $(2k+1)\pi$.

To reduce the contribution of the first factor, one can shorten duration of the drop of phase $\varphi_{RO}(t)$. An example of the pulse formation when drop time is 100 times shorter than the rise time ($T_D = T_R/100$) is shown in Fig. 8. Reduction of the amplitude of the kink is obvious. Also, the pulse shape looks more symmetric. It should be noted that an order-of-magnitude reduction of T_D leads to a nearly 1.5 factor reduction of phase ξ_1, i.e., $\xi_1 = 0.548$.

To exclude directly the contribution of the first factor, one can reduce the coefficient C in Eq. (18) by a factor T_R/T and keep the same ratio between T_R and T_D as in Fig. 7. Comparison of pulses produced by the phase modulation $\varphi_{RO}(t)$ with C (red solid line) and reduced coefficient C (dotted blue line) is shown in Fig. 9. The improvement in the pulse shape is obvious.

To avoid the influence of both factors, we consider the case when durations of the phase rising and dropping stages are reduced by an order of magnitude. Then, time evolution in both stages becomes almost linear in time. Maximum amplitude of the phase excursion is also properly reduced to exclude unbalanced competition of the phases Ωt and $\varphi_{RO}(t)$.

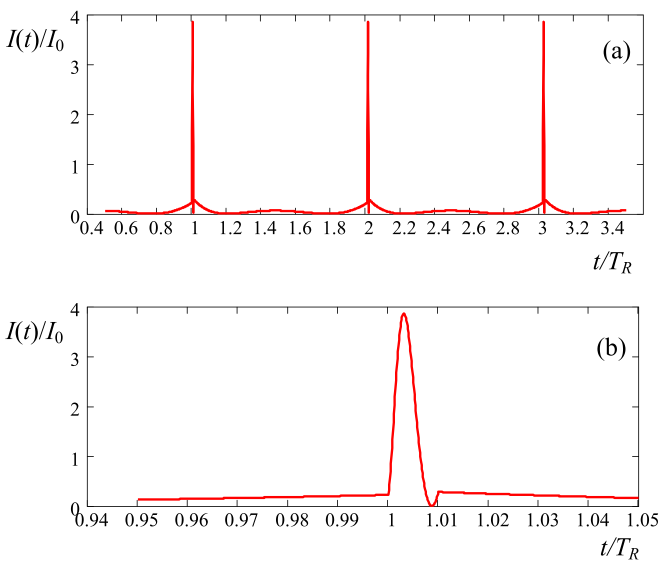

FIG. 8. (a) Sequence of pulses, generated by the frequency filtering, when $T_D = T_R/100$. The time scale is in units of the duration of the phase rising stage, T_R. (b) Zoom in on the area of the pulse formation around $t = T_R$.

Such a modified phase is described by the equation

$$\varphi_{\rm sh}(t) = C_{\rm sh}\sum_{k=0}^{+\infty}\phi_{\rm sh}\left[t - k\left(\frac{T_R+T_D}{10}\right)\right], \tag{27}$$

where

$$\phi_{\rm sh}(t) = \phi_{Rs}(t) + \phi_{Ds}(t), \tag{28}$$

$$\phi_{Rs}(t) = (1 - e^{-t/T_R})\left[\theta(t) - \theta\left(t - \frac{T_R}{10}\right)\right], \tag{29}$$

$$\phi_{Ds}(t) = \left[e^{-(t-\frac{T_R}{10})/T_D} - e^{-1}\right] \times\left[\theta\left(t - \frac{T_R}{10}\right) - \theta\left(t - \frac{T_R+T_D}{10}\right)\right], \tag{30}$$

and $C_{\rm sh} = 2\pi T_R/[(1-e^{-0.1})(T_R+T_D)]$ is reduced by a factor of $T_R/(T_R+T_D)$ excluding unbalanced competition of phases.

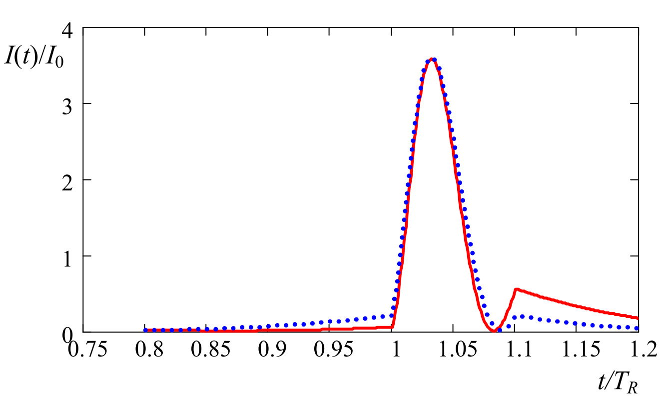

FIG. 9. Comparison of pulses produced by the phase modulation $\varphi_{RO}(t)$ with C (red solid line) and reduced coefficient C (dotted blue line). Durations of the phase rising and dropping stages satisfy the relation $T_D = T_R/10$.

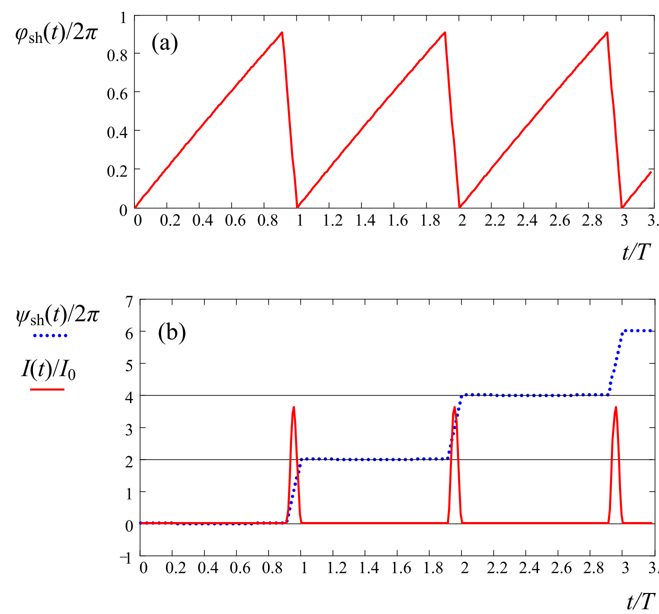

FIG. 10. (a) Phase $\varphi_{\rm sh}(t)$ evolution in time. (b) Evolution of phase $\psi_{\rm sh}(t)$ (blue dotted line) and pulse sequence produced by the filtering of the phase-modulated field (red solid line).

The period of phase modulation is $T = (T_R + T_D)/10$ in this case. Phase modulation and formation of pulses are shown in Fig. 10, demonstrating perfect pulses and desirable evolution of phase $\psi_{\rm sh}(t)$, which is

$$\psi_{\rm sh}(t) = \Omega t + \xi_{1s} - \varphi_{\rm sh}(t). \tag{31}$$

Here, ξ_{1s} is calculated with the help of Eq. (22), where $\varphi_{RO}(t)$ is substituted by $\varphi_{\rm sh}(t)$. As a result of shortening of the phase rise and drop periods, duration of pulses shortens to $T_D/20$.

It can be shown that in contrast to the considered case proportional lengthening of the durations of the phase rising and dropping stages with respect to T_R and T_D deteriorates the shape of the generated pulses.

Two methods of the sawtooth modulation of the field phase (i) by additive synthesis of many harmonics and (ii) by charge and discharge processes allow one to generate short pulses. Duration of the pulses is limited by the highest number of the harmonic in the method (i) (see Sec. III), and its advantage is a clean spectrum defined by the modulation frequency Ω and spectral width of the fundamental frequency. In the method (ii) duration of the pulses is defined by the rate of the discharge process. To my best knowledge the Gunn diode and IMPATT diode are capable to produce discharge with the rate up to 1 THz. Thus, quite short pulses could be produced by the method (ii) (see Refs. [28,29]).

VI. FREQUENCY FILTERING METHODS

Removal of the selected spectral component of the frequency comb with frequency ω_s can be implemented by resonant filters with a single absorption line $L(\omega - \omega_f)$ centered at frequency ω_f. The width of this line, Γ, has to be much smaller than the distance Ω between the frequency components of the comb, i.e., $\Gamma \ll \Omega$.

Details of the application of resonant filters are discussed in Ref. [17]. Below, only the list of conditions, restrictions, and results is given.

For the filter with a homogeneously broadened absorption line, the amplitude of the selected line, which is tuned in exact resonance ($\omega_s = \omega_f$), decreases as $E_{fs} = \exp(-d/2)E_s$, where $d = \alpha_B l$ is the optical thickness of the filter and α_B is Beer's law absorption coefficient. The filtering becomes effective if $d/2 \gg 1$. However, there is a limit set to the optical thickness of the filter by the condition that the spectral components $\omega_s \pm \Omega$ neighboring the selected component ω_s must be unaffected. This condition is satisfied, if $d \ll 4\Omega/\Gamma$, where Γ is a width at half maximum of the Lorentzian absorption line of a single particle in the absorber. If only the selected frequency component is affected by the resonant filter, then Eq. (10), describing in Sec. III the filtered comb in the ideal case of 100% removal and filtering, is modified as

$$E_f(t) = E(t)\left[e^{i\varphi_N(t)} - e_{n_m}(N)(1 - e^{-d/2})e^{in_m\Omega t}\right]. \quad (32)$$

In a similar way, Eq. (24) in Sec. V is modified in the case of the resonant filter.

If the absorption line in the filter is Doppler broadened, then the amplitude of the component filtered by the absorber decreases as $E_{fs} = \exp[-dF_D(0)/2]E_s$, where $F_D(0) = \sqrt{\pi \ln 2}\Gamma/\Delta\omega_D$ and $\Delta\omega_D$ is the Doppler width, which is supposed to be much larger than Γ, for example, $\Gamma/\Delta\omega_D = 10^{-2}$. It seems that the neighboring components are not affected if $\Omega \gg \Delta\omega_D$. However, because for $\Gamma/\Delta\omega_D = 10^{-2}$ and $\Omega > 1.8\Delta\omega_D$ the Voigt profile, which is the convolution of the Lorentzian with the Gaussian describing the Doppler broadening, has Lorentzian wings [30], their influence on the spectral neighbors of the filtered component is negligible if $d \ll 4\Omega/\Gamma$. Thus, effective filtering takes place if $\Omega \gg \Delta\omega_D$ and optical thickness satisfies the condition $1.35\Delta\omega_D/\Gamma \ll d \ll 4\Omega/\Gamma$.

The filtering of the selected spectral component can be also implemented by the method based on a spectral line-by-line pulse shaper (see, for example, Ref. [21]). A many-pixel LCM array allows one in this technique to control both amplitude and phase of individual spectral lines of the field with a comb spectrum. The LCM can be tuned such that only the selected spectral line of the comb is suppressed.

An effective and flexible method of creating nanosecond pulses can be implemented by filtering of the frequency comb through laser-cooled atoms with a modest optical depth. For example, a D_1-line transition ($\lambda = 795$ nm) of ^{85}Rb atoms in a two-dimensional magneto-optical trap has an almost homogeneous width $\Gamma \approx 6$ MHz (see, for example, Ref. [31]). Therefore, with the modulation frequency of EOM $\Omega = 30$ MHZ one can generate 1.5-ns pulses for the sawtooth phase modulation I with $N = 5$ and 1.24 ns for the sawtooth phase modulation II with $T_D = T_R/10$ by the filtering through the cloud of laser-cooled ^{85}Rb atoms. For $N = 10$, pulse duration shortens to 800 ps. If the modulation frequency is increased to $\Omega = 300$ MHZ, then the duration of the generated pulses is shortened to 150 ps for $N = 5$ and to 80 ps for $N = 10$. For the sawtooth phase modulation II with $T_D = T_R/10$ pulses shorten to 124 ps.

For a frequency filter one can use a vapor of ^{87}Rb atoms. Assume that the selected frequency of the comb is tuned in resonance with the $S_{1/2}, F = 1 \rightarrow P_{1/2}, F = 2$ transition of the D_1 line of natural Rb ($\lambda = 795$ nm). Below, we take the parameters of the experiment [32] where spectral properties of the electromagnetically induced transparency were studied in this vapor. The natural linewidth of the Rb D_1 line is $\Gamma = 5.4$ MHz and Doppler broadening is $\Delta\omega_D = 500$ MHz. Selecting the phase modulation frequency $\Omega = 10$ GHz, which is 20 times larger than the Doppler width $\Delta\omega_D = 500$ MHz, we satisfy the condition $\Omega \gg \Delta\omega_D$. According to the estimates given in Ref. [17] for the Rb cell with the length $l = 5$ cm and atomic density $N_1 = 6 \times 10^{10}$ cm^{-3}, the modification of the spectral components neighboring the selected one is almost negligible. For this atomic density the effective optical depth of the cell at the selected line center is $dF_D(0) = 14.4$ while $d = 905$. With these values of the parameters Ω, $\Delta\omega_D$, Γ, and d, the condition $1.35\Delta\omega_D/\Gamma \ll d \ll 4\Omega/\Gamma$ is easily satisfied.

For the modulation frequency $\Omega = 10$ GHz, filtering through the atomic vapor or removing the selected spectral component with the help of the LCM [21] produces much shorter pulses. For example, for the sawtooth phase modulation I with $N = 5$ and sawtooth phase modulation II with $T_D = T_R/10$ one can generate 4.5- and 3.7-ps pulses, respectively. For the sawtooth phase modulation I consisting of ten harmonics ($N = 10$), duration of the pulses shortens to 2.4 ps. If the number of the harmonics increases to $N = 50$, pulse duration shortens to 495 fs.

For a selective filter one can use organic molecules doped in a polymer matrix. It is experimentally possible to burn a broad spectral hole in their spectrum with a sharp absorption peak sitting at its center. Such a structure is persistent at liquid helium temperature. The frequency resolution of the persistent spectral hole burning is limited by the width of the homogeneous zero-phonon line of the chromophore molecules, which typically has a width of 10^{-2}–10^{-4} cm^{-1} or less [33,34]. The holes could be burned in a planar waveguide geometry where a thin polymer film with doped molecules is superimposed as a cover layer on a planar glass waveguide [35,36]. Then, illumination in the transverse direction with low absorption creates a hole, while a weak probing field propagates in a longitudinal wave guiding direction with high absorption. For example, such a waveguide with a spectral hole acting as subgigahertz narrow-band filter was proposed to observe slow light phenomena in Refs. [37,38].

VII. CONCLUSION

The methods of pulse shaping with the help of harmonic phase modulation and subsequent removal of the selected spectral component (i) by a resonant filter or (ii) by control of phases of several spectral components have many advantages. Because of small losses and an easy method to group pulses, the method (i) can be used to shape single-photon wave packets, creating time-bin qubits and qutrits. However, pulse duration in the method (i) is limited since by elevating the modulation index sufficiently for pulse shortening the contrast between the pulse amplitude and the pedestal decreases. By the method (ii) one can produce very short pulses with a good amplitude-pedestal contrast. However, large losses inherent in this method do not allow one to work with single-photon fields.

Pulse shaping by the removal of the selected spectral component of the sawtooth phase-modulated cw field works with the fixed modulation index of moderate value. Short pulses are generated during fast dropping of the phase. The faster this drop is, the shorter the pulse is formed. Its duration can be made an order or two orders of magnitude shorter than the phase modulation period. The contrast between the pedestal and the pulse maximum increases with increasing of the rate of the phase drop. Frequency synthesis of many harmonics or charge and fast discharge can be used to produce sawtooth wave forms, which feed EOMs, producing a sawtooth phase-modulated field. Both methods of producing sawtooth wave forms are easily available in modern electronics. Therefore, the method proposed in this paper is applicable to shape single-photon fields of short duration with low losses and can compete with the method (ii) to shape pulses of classical fields.

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[1] A. M. Weiner, *Ultrafast Optics* (Wiley, New York, 2009).
[2] J. C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena*, 2nd ed. (Academic, San Francisco, 2006).
[3] C. W. Hillegas, J. X. Tull, D. Goswami, D. Strickland, and W. S. Warren, Opt. Lett. **19**, 737 (1994).
[4] M. R. Fetterman, D. Goswami, D. Keusters, W. Yang, J.-K. Rhee, and W. S. Warren, Opt. Express **3**, 366 (1998).
[5] F. Verluise, V. Laude, Z. Cheng, Ch. Spielmann, and P. Tournois, Opt. Lett. **25**, 575 (2000).
[6] E. B. Treacy, Phys. Lett. A **28**, 34 (1968).
[7] D. Grischkowsky, Appl. Phys. Lett. **25**, 566 (1974).
[8] J. E. Bjorkholm, E. H. Turner, and D. B. Pearson, App. Phys. Lett. **26**, 564 (1975).
[9] H. Nakatsuka, D. Grischkowsky, and A. C. Balant, Phys. Rev. Lett. **47**, 910 (1981).
[10] B. Nikolaus and D. Grischkowsky, Appl. Phys. Lett. **42**, 1 (1983).
[11] M. T. Loy, Appl. Phys. Lett. **26**, 99 (1975).
[12] M. T. Loy, IEEE J. Quantum Electron. **QE-13**, 388 (1977).
[13] S. Hisatake, Y. Nakase, K. Shibuya, and T. Kobayashi, Opt. Lett. **30**, 777 (2005).
[14] A. M. Weiner, Opt. Commun. **284**, 3669 (2011), special issue on optical pulse shaping, arbitrary wave form generation, and pulse characterization.
[15] F. Vagizov, V. Antonov, Y. V. Radeonychev, R. N. Shakhmuratov, and O. Kocharovskaya, Nature (London) **508**, 80 (2014).
[16] R. N. Shakhmuratov, F. G. Vagizov, V. A. Antonov, Y. V. Radeonychev, M. O. Scully, and O. Kocharovskaya, Phys. Rev. A **92**, 023836 (2015).
[17] R. N. Shakhmuratov, Phys. Rev. A **95**, 033805 (2017).
[18] I. R. Khairulin, V. A. Antonov, Y. V. Radeonychev, and O. A. Kocharovskaya, J. Phys. B **51**, 235601 (2018).
[19] J. Brendel, N. Gisin, W. Tittel, and H. Zbinden, Phys. Rev. Lett. **82**, 2594 (1999).
[20] I. Marcikic, H. de Riedmatten, W. Tittel, V. Scarani, H. Zbinden, and N. Gisin, Phys. Rev. A **66**, 062308 (2002).
[21] Z. Jiang, D. E. Leaird, and A. M. Weiner, IEEE J. Quantum Electron. **42**, 657 (2006).
[22] K. Fong, M. Laverty, S. Fang, and W. Uzat, in *Proceedings of the 1997 Particle Accelerator Conference (Cat. No. 97CH36167)* (IEEE, Piscataway, NJ, 1997), Vol. 3, pp. 3057–3059.
[23] B. Bruneau, T. Gans, D. O’Connell, A. Greb, E. V. Johnson, and J.-P. Booth, Phys. Rev. Lett. **114**, 125002 (2015).
[24] R. N. Shakhmuratov, F. G. Vagizov, M. O. Scully, and O. Kocharovskaya, Phys. Rev. A **94**, 043849 (2016).
[25] R. N. Shakhmuratov, F. G. Vagizov, and O. Kocharovskaya, Phys. Rev. A **84**, 043820 (2011).
[26] R. N. Shakhmuratov, Phys. Rev. A **85**, 023827 (2012).
[27] S. Agnello, R. Boscaino, M. Cannas, F. M. Gelardi, and R. N. Shakhmuratov, Phys. Rev. A **59**, 4087 (1999).
[28] V. Gruzinskis, J. H. Zhao, P. Shiktorov, and E. Starikov, Material Science Forum **297–298**, 341 (1999).
[29] T. Misawa, Semicond. Semimet. **7**, 371 (1971).
[30] R. N. Shakhmuratov and J. Odeurs, Phys. Rev. A **78**, 063836 (2008).
[31] J. F. Chen, H. Jeong, L. Feng, M. M. T. Loy, G. K. L. Wong, and S. Du, Phys. Rev. Lett. **104**, 223602 (2010).
[32] M. D. Lukin, M. Fleischhauer, A. S. Zibrov, H. G. Robinson, V. L. Velichansky, L. Hollberg, and M. O. Scully, Phys. Rev. Lett. **79**, 2959 (1997).
[33] H. Schwoerer, D. Erni, and A. Rebane, J. Opt. Soc. Am. B **12**, 1083 (1995).
[34] A. Renn, U. P. Wild, and A. Rebane, J. Phys. Chem. **106**, 3045 (2002).
[35] M. Tschanz, A. Rebane, and U. P. Wild, Opt. Eng. **34**, 1936 (1995).
[36] M. Tschanz, A. Rebane, D. Reiss, and U. P. Wild, Mol. Cryst. Liq. Crust. **283**, 43 (1996).
[37] R. N. Shakhmuratov, A. Rebane, P. Megret, and J. Odeurs, Phys. Rev. A **71**, 053811 (2005).
[38] A. Rebane, R. N. Shakhmuratov, P. Megret, and J. Odeurs, J. Lumin. **127**, 22 (2007).